UAV Trajectory Planning and Tracking for User Localization in Wireless Networks

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***Abstract*—In this paper, we study UAV-aided user localization in wireless networks without assuming perfect knowledge of the UAV position. The problem is therefore formulated as joint user localization and UAV tracking. We exploit time-of-arrival (ToA) and received signal strength (RSS) measurements collected by a UAV and propose an Expectation–Maximization-based least-squares simultaneous localization and mapping (SLAM) framework to classify measurements as line-of-sight or non-line-of-sight, learn the radio channel, and jointly localize users and track the UAV. The framework incorporates auxiliary measurements from onboard GPS and inertial measurement unit (IMU) sensors to improve accuracy. In addition, UAV trajectory optimization is performed to further enhance localization performance. Simulation results demonstrate superior performance compared to existing approaches.**

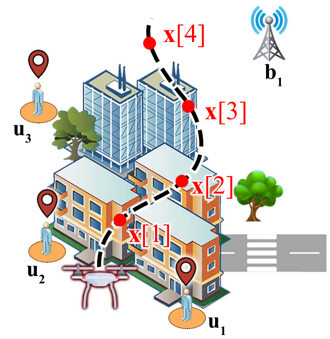


Fig. 1: UAV-aided ground user localization system.

I. Introduction

In a wireless localization system, nodes with known positions, referred to as anchor nodes (stationary or mobile), collect radio measurements from users radio frequency (RF) signals, enabling localization. Common measurements include received signal strength (RSS), time-of-arrival (ToA), angle-of-arrival (AoA), and time-difference-of-arrival (TDoA) [1].

Recent advances in robotics, miniaturized wireless systems, and AI-guided motion have enabled flying radio access networks (FRANs), where unmanned aerial vehicles (UAVs) equipped with BSs or relays provide rapid wireless connectivity [2]. This mobility allows UAV-mounted BSs to act as mobile anchors with enhanced spatial coverage compared to static anchors. However, the UAV's own position may be uncertain, leading to a joint problem of user localization and UAV tracking [3].

Localization using aerial UAV anchors has gained significant attention recently [4]–[7]. Compared to static anchors, UAVs exploit 3D mobility to collect measurements from multiple spatial locations, effectively acting as virtual static anchors and improving localization performance.

Several works have studied ground user localization relying on RSS and ToA measurements collected by UAV anchors, often assuming perfect UAV position knowledge [3], [4]. UAV trajectory optimization for localization performance has been investigated in various contexts [7], [8], while multi-UAV systems and AI-based approaches are surveyed in recent UAV network literature [2]. Hybrid measurement strategies combining ToA, AoA, and RSS have also been studied for enhanced accuracy. The impact of antenna pattern and urban propagation effects on UAV-based localization provides further insights into practical performance.

In this paper, we propose a joint framework for user localization and UAV tracking using ToA and RSS measurements. We develop an Expectation–Maximization (EM)-based localization algorithm within a least-squares simultaneous localization and mapping (SLAM) framework that jointly classifies measurements as line-of-sight (LoS) or non-line-of-sight (NLoS), learns the radio channel, localizes users, and tracks the UAV. Auxiliary measurements from onboard GPS and inertial measurement unit (IMU) sensors are incorporated to improve accuracy.

To the best of our knowledge, joint user localization and mobile aerial anchor tracking using ToA and RSS measurements has not been previously studied. The main contributions of this work are summarized as follows:

- A mobile-anchor-based localization algorithm exploiting ToA and RSS measurements is proposed.
- A multi-modal least-squares SLAM framework is formulated for joint measurement classification, user localization, and mobile anchor tracking.
- UAV trajectory optimization is incorporated to further enhance localization accuracy.

II. System Model and Problem Formulation

We consider a scenario similar to that illustrated in Fig. 1, where M static terrestrial base stations (BSs) and a UAV relay measure radio signals from K ground users within a given service area. The terrestrial BSs are connected to both the UAV relay and the ground users. We assume that the locations of the static BSs are known, with the m-th BS located at $\mathbf{b}_m =$

$[x_m, y_m, z_m]^{\mathrm{T}} \in \mathbb{R}^3$, $m \in [1, M]$. The K users are distributed over the area, with the k-th user's location denoted by $\mathbf{u}_k = [x_k, y_k]^{\mathrm{T}} \in \mathbb{R}^2$, $k \in [1, K]$. The users are assumed to be static, and their locations are unknown. The main goal of this work is to estimate the user locations based on the radio measurements collected by the UAV relay and the BSs.

A. UAV Model

We assume that the UAV mission lasts for a duration of T seconds. The mission time is discretized into N equal time steps, with the UAV velocity assumed constant within each step. At the n-th time step, the UAV position is denoted by $\mathbf{x}[n] = [x[n], y[n], z[n]]^{\mathrm{T}} \in \mathbb{R}^3$, $n \in [1, N]$. The UAV is equipped with a GPS receiver, and the measured location is given by $\hat{\mathbf{x}}[n] = \mathbf{x}[n] + \mathbf{e}_x$, where $\mathbf{e}_x$ represents GPS measurement noise, modeled as a Gaussian random variable $\mathcal{N}(0, \sigma_{gps}^2 * \mathbf{I}_2)$, with $\mathbf{I}_2$ denoting the 2×2 identity matrix.

The UAV velocity, obtained from an onboard IMU, is denoted by $\hat{\mathbf{v}}[n] = \mathbf{v}[n] + \mathbf{e}_v$, where $\mathbf{v}[n]$ is the true UAV velocity at step n, and $\mathbf{e}_v$ is the velocity measurement noise, modeled as $\mathcal{N}(0, \sigma_{vel}^2 * \mathbf{I}_2)$. Assuming constant speed within each time step, the UAV velocity can then be computed as follows:

$$\mathbf{v}[n] = \frac{\mathbf{x}[n] - \mathbf{x}[n-1]}{\Delta t}, \tag{1}$$

where $\Delta t = \frac{T}{N}$. Therefore, the UAV velocity can be reformulated as

$$\hat{\mathbf{v}}[n] = \frac{\mathbf{x}[n] - \mathbf{x}[n-1]}{\Delta t} + \mathbf{e}_v. \tag{2}$$

B. Channel Model

We now describe the radio channel model between the BS, UAV and ground users. BSs and the UAV-relay have access to two types of radio measurements, namely, RSS (or channel gain) and ToA.

Classically, the channel gain (in dB) between UAV position $\mathbf{x}[n]$ and user location $\mathbf{u}_k$ can be modeled as

$$\hat{g}_k[n] = \begin{cases} g_{k,\mathrm{LoS}}[n] + \eta_{\mathrm{LoS}} & \text{if LoS} \\ g_{k,\mathrm{NLoS}}[n] + \eta_{\mathrm{NLoS}} & \text{if NLoS,} \end{cases} \tag{3}$$

where $g_{k,s}[n] \triangleq \beta_s + \alpha_s \, \phi_k[n]$, where $\phi_k[n] = \log_{10} \|\mathbf{x}[n] - \mathbf{u}_k\|$, and α_s is the path loss exponent, β_s is the average channel gain at the reference point $d = 1$ meter, η_s denotes the shadowing component which is modeled as a Gaussian random variable with $\mathcal{N}(0, \sigma_s^2)$, and finally $s \in \mathcal{S} \triangleq \{\mathrm{LoS}, \mathrm{NLoS}\}$ emphasizes the strong dependence of the propagation parameters on LoS or NLoS scenarios. Note that (3) represents the channel gain which is averaged over the small scale fading of unit variance. We assume that the channel parameters in (3) are not known and need to be estimated from the measurements. The probability distribution of a single measurement in (3) is given by

$$p(g_k[n]) = (f_{k,\mathrm{LoS}}[n])^{w_k[n]} (f_{k,\mathrm{NLoS}}[n])^{(1-w_k[n])}, \tag{4}$$

where $\omega_k[n] \in \{0, 1\}$ is the classifier binary variable (unknown) indicating whether a measurement falls into the LoS or NLoS category, and $f_{k,s}[n]$ has a Gaussian distribution with $\mathcal{N}(\beta_s + \alpha_s \, \phi_k[n], \sigma_s^2)$.

The ToA measurement between the UAV at time step n and the k-th user is modeled as follows

$$\hat{\tau}_k[n] = \begin{cases} \tau_k[n] + \xi_{\mathrm{LoS}} & \text{if LoS} \\ \tau_k[n] + \xi_{\mathrm{NLoS}} & \text{if NLoS,} \end{cases} \tag{5}$$

where $\tau_k[n] = \frac{\|\mathbf{x}[n] - \mathbf{u}_k\|}{c}$, c is the speed of light, and ξ_s is the timing measurement noise which can be modeled as a Gaussian random variable with $\mathcal{N}(\mu_{\tau,s}, \sigma_{\tau,s}^2)$. Note that the measurement noise is different for each segments of LoS/NLoS. $\mu_{\tau,s}$ reflects the error when there is no LoS link between the receiver and the transmitter, and is considered zero when $s = \mathrm{LoS}$. The $\sigma_{\tau,s}^2$ similarly depends on the link status and also on the Bandwidth of the channel. Generally speaking, $\sigma_{\tau,s}^2 \propto \frac{1}{\text{Bandwidth}} \frac{1}{\mathrm{SNR}_{k,s}[n]}$. Where $\mathrm{SNR}_{k,s}[n]$ is the signal-to-noise-ratio (SNR) of the signal between the UAV at time step n and user k at segment s. For simplicity, in this paper we assume an average SNR at each segment to model the ToA as follows

$$\sigma_{\tau,s}^2 \propto \frac{1}{\text{Bandwidth}} \frac{1}{\overline{\mathrm{SNR}}_s}, \tag{6}$$

where $\overline{\mathrm{SNR}}_s$ is the average SNR at each segment of LoS/NLoS. The probability distribution of a measurement in (5) is given by

$$p(\tau_k[n]) = (f_{\tau,k,\mathrm{LoS}}[n])^{w_k[n]} (f_{\tau,k,\mathrm{NLoS}}[n])^{(1-w_k[n])}, \tag{7}$$

where $f_{\tau,k,s}[n]$ has a Gaussian distribution with $\mathcal{N}(\frac{\|\mathbf{x}[n] - \mathbf{u}_k\|}{c} + \mu_{\tau,s}, \sigma_{\tau,s}^2)$. It is worth mentioning that, the same classification variable $w_k[n]$, as used to classify measurements in (4), is utilized to classify ToA measurements in (7). This stems from the fact that both measurements are taken at the same time and the status of the link is the same for both types of measurements.

The channel gain and ToA measurements between user k and BS m represented by $\hat{g}_{m,k}, \hat{\tau}_{m,k}$, and the channel gain and ToA measurements between the UAV at time step n and BS m are denoted by $\hat{g}_m[n], \hat{\tau}_m[n]$, respectively. The above observations follow similar link model akin to (3) and (5). The probability distribution of measurements collected by BSs from the UAV and the users follow similar model in (4) and (7).

III. User Localization and UAV Tracking

In this section, we propose an algorithm to estimate the user locations from the radio measurements collected by the UAV and BSs. Let us denote an arbitrary set of measurements available at UAV (taken by the UAV and also by the BSs) during the mission as $\mathcal{G} = \{o[n], \gamma_k[n], \gamma_m[n], \gamma_{m,k}; \forall n, m, k\}$, where $o[n]$ is the UAV odometry measurements at time step n, $\gamma_k[n]$ is a tuple of measurements collected by the UAV at time step n from user k, $\gamma_m[n]$ is a tuple of measurements collected by BS m from the UAV at time step n, and $\gamma_{m,k}$ is a tuple of measurements collected by BS m from user k defined as follows

$$\begin{aligned} o[n] &\triangleq (\hat{\mathbf{x}}[n], \hat{\mathbf{v}}[n]), n \in [1, N], \\ \gamma_k[n] &\triangleq (\hat{g}_k[n], \hat{\tau}_k[n])\,, n \in [1, N], k \in [1, K] \\ \gamma_m[n] &\triangleq (\hat{g}_m[n], \hat{\tau}_m[n])\,, n \in [1, N], m \in [1, M], \\ \gamma_{m,k} &\triangleq (\hat{g}_{m,k}, \hat{\tau}_{m,k})\,, m \in [1, M], k \in [1, K]. \end{aligned}$$

Note that the LoS/NLoS status of channel gain and ToA measurements is not available and needs to be estimated as well. It is also worth mentioning that, the true location and the velocity of the UAV are not available (i.e. the GPS and the IMU measurements are subject to the noise), therefore we not only have to localize the users but also need to track the UAV location.

Assuming that collected measurements conditioned on the channel and user positions are independent and identically distributed (i.i.d), the negative log-likelihood of measurements can be written as

$$\mathcal{L} = \mathcal{L}_{\text{UAV}} + \mathcal{L}_{\text{UAV-UE}} + \mathcal{L}_{\text{BS-UAV}} + \mathcal{L}_{\text{BS-UE}}, \tag{8}$$

where $\mathcal{L}_{\text{UAV}} = \ell(o[n]; \forall n)$ is the loss function derived from the log-likelihood of the odometry measurements, where $\ell(.)$ indicates the log-likelihood operation. $\mathcal{L}_{\text{UAV-UE}} = \ell(\gamma_k[n]; \forall n, k)$ is the loss function derived from the log-likelihood of measurements taken by the UAV from users. Similarly, $\mathcal{L}_{\text{BS-UAV}} = \ell(\gamma_m[n]; \forall m, n)$ is the loss function derived from the log-likelihood of measurements taken by BSs from the UAV, and $\mathcal{L}_{\text{BS-UE}} = \ell(\gamma_{m,k}; \forall m, k)$ is the loss function derived from the log-likelihood of measurements taken by BSs from the users.

The estimate of the unknown user and the UAV locations can then be obtained by solving

$$\min_{\substack{\mathbf{x}[n],\, \mathbf{u}_k, \theta, \mathcal{W} \\ \forall n,k}} \quad \mathcal{L}, \tag{9}$$

where $\theta = \left\{\alpha_{\text{s}}, \beta_{\text{s}}, \sigma_s^2, \sigma_{\tau,s}^2, \mu_{\tau,s}; s \in \{\text{LoS}, \text{NLoS}\}\right\}$, and $\mathcal{W} = \{\omega_k[n], \omega_m[n], \omega_{m,k}; \forall n, m, k\}$. Solving problem (9) is challenging, since it is a mixed-integer problem of simultaneous user localization, UAV tracking, channel learning, and measurements classification. In the following sections, we introduce an iterative algorithm to jointly classify the measurements and learn the channel parameters, and at the same time localize the users and track the UAV. At each iteration of the algorithm we employ an EM algorithm to classify the measurements and learn the channel, and then we use a least-square-based SLAM framework to localize the users and track the UAV.

A. Learning and Classification

At this phase of the algorithm, we fix the user and the UAV location and we solve the problem (9) to find the classification variables and also to learn the channel. Moreover, since the classification variables are the same for both channel gain and the ToA measurements, without loss of optimally, we classify the measurements only considering the channel gains. For the sake of clarity, let's only consider the channel gain measurements between the UAV and the users. Therefore, the log likelihood of the channel gain measurements can be rewritten as follows

$$\mathcal{L}^g_{\text{UAV-UE}} = \sum_{n,k} \log p\left(\hat{g}_k[n]; \theta^g\right) \tag{10}$$

where $\mathcal{L}^g_{\text{UAV-UE}}$ indicates the log-likelihood for only channel gain measurements between the UAV and the users, and $\theta^g = \left\{\alpha_{\text{s}}, \beta_{\text{s}}, \sigma_s^2; s \in \{\text{LoS}, \text{NLoS}\}\right\}$. For each measurement, let $q(w_k[n])$ be a distribution over $w_k[n]$. We can then reformulate (10) as

$$\mathcal{L}_{\text{UAV-UE}} \geq \sum_{n,k} \sum_{w_k[n] \in \mathcal{S}} q(w_k[n]) \log \frac{p(\hat{g}_k[n], w_k[n]; \theta^g)}{q(w_k[n])}, \tag{11}$$

where the inequality follows from Jensen's inequality which is a lower bound on the original likelihood. The lower bound can be tighten by choosing q as follows

$$q(w_k[n] = s) = \frac{p(\hat{g}_k[n] |\, s; \theta^g)\pi_s}{\sum_{s' \in \mathcal{S}} p(\hat{g}_k[n] |\, s'; \theta^g)\pi_{s'}}, \tag{12}$$

Let's denote $\Omega_{k,s}[n] = q(w_k[n] = s)$, therefor the maximum log-likelihood estimation of parameters θ^g can be obtained by solving the following problem

$$\max_{\theta^g, \pi_s} \sum_{n,k} \sum_{s \in \mathcal{S}} \Omega_{k,s}[n] \log \frac{p(\hat{g}_k[n] |\, s; \theta^g)\pi_s}{\Omega_{k,s}[n]} \tag{13a}$$

$$\text{s.t.} \sum_{s \in \mathcal{S}} \pi_s = 1. \tag{13b}$$

This problem is non-convex, therefore, we find a sub-optimal solution by solving (13) iteratively. This algorithm iterates between two steps known as expectation and maximization (E-M steps). During the E-step, the $\Omega_{k,s}[n]$ is computed according to (12) while fixing parameters θ^g. In the M-step, problem (13) is solved only for parameters θ^g by fixing $\Omega_{k,s}[n]$. We denote i as the iteration index of the E-M algorithm and the process is repeated for I iterations.

Let $\theta^{g,(i-1)}, \pi_{s'}^{(i-1)}$ be the parameters available from the $(i-1)$-th iteration, then during the E-step we have

$$\Omega_{k,s}^{(i)}[n] = \frac{p(\hat{g}_k[n] |\, s; \theta^{g,(i-1)})\pi_s^{(i-1)}}{\sum_{s' \in \mathcal{S}} p(\hat{g}_k[n] |\, s'; \theta^{g,(i-1)})\pi_{s'}^{(i-1)}}. \tag{14}$$

For the M-step, (13) can be reformulated as follows:

$$\max_{\theta^{g,(i)}, \pi_s^{(i)}} \sum_{n,k} \sum_{s \in \mathcal{S}} \Omega_{k,s}^{(i)}[n] \log \frac{p(\hat{g}_k[n] |\, s; \theta^{g,(i)})\pi_s^{(i)}}{\Omega_{k,s}^{(i)}[n]} \tag{15a}$$

$$\text{s.t. (13b).} \tag{15b}$$

The objective function (15a) is concave. By setting the derivative of (15) with respect to $\theta^{g,(i)}, \pi_s^{(i)}$ to zero and accounting for the constraint, we can solve this problem. We skip the details of the solution for the sake of brevity.

We now continue with the ToA measurements. The $\sigma_{\tau,s}^2, \mu_{\tau,s}; s \in \{\text{LoS}, \text{NLoS}\}$ can be found as follows

$$\min_{\substack{\sigma^2_{\tau,s},\mu_{\tau,s}\\ s\in\mathcal{S}}} \quad \mathcal{L}^{\tau}_{\text{UAV-UE}}, \tag{16}$$

where $\mathcal{L}^{\tau}_{\text{UAV-UE}}$ indicates the log-likelihood for only ToA measurements between the UAV and the users. We can solve (16) by setting the derivative of the objective function in (16) with respect to $\sigma^2_{\tau,s}, \mu_{\tau,s}$ to zero.

Finally, to obtain the labels for each measurement, we user the hard classification as follows

$$w_k[n] = \begin{cases} 1 & \text{if } \Omega^{(I)}_{k,s} > 0.5 \\ 0 & \text{else} \end{cases}. \tag{17}$$

Similar to above analysis, the E-M algorithm can be solved for $\mathcal{L}$ defined in (8) by considering all the collected measurements to estimate all classification variables $\mathcal{W}$.

B. User Localization and UAV Tracking

Once we know the classification variables $\mathcal{W}$, we continue to find the location of the users and track the UAV. The optimization (9) can now rewritten as

$$\min_{\substack{\mathbf{x}[n],\,\mathbf{u}_k\\ \forall n,k}} \quad \mathcal{L}. \tag{18}$$

Solving this problem is still challenging since the objective function is non-linear and non-convex. To deal with this problem, we employ an iterative approach similar to the one presented in [9], where at each iteration the problem is first locally linearized and then is solved. The algorithm then iterates until convergence. Let's assume that we want to optimize the following problem

$$\min_{\vartheta} \quad \sum_i \mathbf{e}_i^T(\vartheta_i)\mathbf{Q}_i^{-1}\mathbf{e}_i(\vartheta_i). \tag{19}$$

where $\vartheta = [\vartheta_0^T, \vartheta_1^T, \cdots]^T$ is a vector of all the unknown variables, $\mathbf{e}_i(\vartheta_i)$ is a vector function of the unknown variables ϑ_i, and $\mathbf{Q}_i$ is a known diagonal matrix. By using the first-order Taylor approximation around an initial guess $\breve{\vartheta}_i$, we can write

$$\mathbf{e}(\breve{\vartheta}_i + \Delta\vartheta_i) \approx \breve{\mathbf{e}}_i + \mathbf{J}_i\Delta\vartheta_i \tag{20}$$

where $\breve{\mathbf{e}}_i \triangleq \mathbf{e}(\breve{\vartheta}_i)$, and $\mathbf{J}_i$ is the Jacobian of $\mathbf{e}_i(\vartheta_i)$ computed in $\breve{\vartheta}_i$. By substituting (20) in (19), we have

$$\min_{\vartheta} \quad \sum_i \breve{\mathbf{e}}_i^T\breve{\mathbf{e}}_i + 2\breve{\mathbf{e}}_i^T\mathbf{Q}_i^{-1}\mathbf{J}_i\Delta\vartheta_i + \Delta\vartheta_i^T\mathbf{J}_i^T\mathbf{Q}_i^{-1}\mathbf{J}_i\Delta\vartheta_i. \tag{21}$$

We can reformulate (21) in a matrix form as follows

$$\min_{\vartheta} \quad \breve{\mathbf{e}} + 2\mathbf{b}^T\Delta\vartheta + \Delta\vartheta^T\,\mathbf{H}\,\Delta\vartheta, \tag{22}$$

where $\breve{\mathbf{e}} \triangleq [\breve{\mathbf{e}}_0^T, \breve{\mathbf{e}}_1^T, \cdots]^T$, $\mathbf{b} = [\breve{\mathbf{e}}_0^T\mathbf{Q}_0^{-1}\mathbf{J}_0, \breve{\mathbf{e}}_1^T\mathbf{Q}_1^{-1}\mathbf{J}_1, \cdots]^T$, and $\mathbf{H}$ is a block diagonal matrix defined as

$$\mathbf{H} \triangleq \operatorname{diag}\left(\mathbf{J}_0^T\mathbf{Q}_0^{-1}\mathbf{J}_0, \mathbf{J}_1^T\mathbf{Q}_1^{-1}\mathbf{J}_1, \cdots\right). \tag{23}$$

The linear problem (21) can now be solved and the solution is given by $\vartheta^* = \breve{\vartheta} + \Delta\vartheta^* = \breve{\vartheta} - \mathbf{H}^{-1}\,\mathbf{b}$, where $\breve{\vartheta} = [\breve{\vartheta}_1^T, \breve{\vartheta}_2^T, \cdots]^T$ is a vector of initial guesses. This procedure then repeats until ϑ^* converges to a local minima.

With some mathematical manipulations and akin to [3], our original optimization problem (18) can be reformulated in the form of (19), and then solved using the aforementioned iterative algorithm. The details of reformulating (18) in the form of (19) are skipped due to the limited space.

The different steps of the proposed algorithm are briefly described in Algorithm 1.

Algorithm 1 User localization, UAV tracking, measurement classification, and channel learning algorithm

1: Given all measurements set $\mathcal{G}$.
2: Initialize $\mathcal{W}, \theta$, users' locations, and UAV location estimates
3: 1) **E-M algorithm**:
4: Fixing users and UAV locations estimates, find classification variables $\mathcal{W}$, and the channel parameters θ using the EM algorithm.
5: 2) **Least-squares-based SLAM**:
6: Solve (18) given estimated $\mathcal{W}, \theta$.
7: Go to Step 3, until the change on UAV and users' locations estimate is small.

IV. UAV Trajectory Optimization

Unlike static anchors, a UAV mobile anchor can optimize its trajectory to improve localization performance. The path planning problem can thus be formulated to identify feasible trajectories, under time or energy constraints, that maximize the informativeness of the collected measurements. This formulation is commonly referred to as active learning [10].

In localization, NLoS measurements degrade accuracy due to shadowing and biased timing, while maintaining LoS links to all users throughout the mission is generally infeasible. Consequently, the UAV trajectory must balance the collection of informative LoS measurements with limited mission time.

We address this problem using the Fisher information matrix (FIM) [11], which quantifies the information content of measurements about unknown user locations. By exploiting the structure of the FIM, we derive a greedy path planning for data collection. We focus exclusively on UAV-based ToA measurements and ignore measurements from base stations.

A. Cramér-Rao Bound Analysis

According to the Cramér-Rao bound (CRB) [12], the mean squared error (MSE) of the estimated parameters $\mathbf{u}$ given the measurements for an unbiased estimator is lower bounded by $\text{MSE}(\mathbf{u}) \geq \operatorname{tr}(\mathbf{F}_N^{-1})$, where $\operatorname{tr}(\mathbf{F}_N^{-1})$ is the trace of $\mathbf{F}_N^{-1}$, and $\mathbf{F}_N$ is the FIM for all measurements collected up to time step N is given by $\mathbf{F}_N = \mathbf{F}_{N-1} + \sum_{k=1}^{K}\mathbf{H}_k[N]$, where

$$\mathbf{H}_k[n] = w_k[n]\mathbf{H}_{k,\text{LoS}}[n] + (1 - w_k[n])\mathbf{H}_{k,\text{NLoS}}[n], \tag{24}$$

and $\mathbf{H}_{k,s}[n]$ can be derived akin to the work in [13].

Using the matrix inversion lemma, it can be shown that $\mathbf{F}^{-1}_{N,s}$ follows a recursive relation as follows $\mathbf{F}_N^{-1} = \mathbf{F}_1^{-1} -$

$\sum_{n=2}^{N} \mathbf{R}[n]$, where $\mathbf{R}[n]$ is defined as the amount of improvement in the estimation within time slot n which is given by

$$\mathbf{R}[n] = \mathbf{F}_{n-1}^{-1} \left(\sum_{k=1}^{K} \mathbf{H}_k[n]^{-1} + \mathbf{F}_{n-1}^{-1} \right)^{-1} \mathbf{F}_{n-1}^{-1}. \tag{25}$$

This recursion will be used to optimize the UAV trajectory.

B. Greedy Trajectory Design

In this section, we propose a greedy approach where the trajectory is devised locally and piece by piece. At each time step, the UAV aims to find the best location for the next step to go and collect measurements, potentially offering the maximum improvement in the estimation error. We define the objective function in the greedy algorithm at time step n as

$$L(\mathbf{x}[n]) = \begin{cases} \operatorname{tr}(\hat{\mathbf{R}}_{n|n-1}), & \|\mathbf{x}[n] - \mathbf{x}_{\mathrm{F}}\| \leq \lambda \\ -\infty, & \text{otherwise} \end{cases}, \tag{26}$$

where $\hat{\mathbf{R}}_{n|n-1}$ is the estimation of $\mathbf{R}[n]$ given $\hat{\mathbf{u}}$ up to step n, and $\lambda = D_{\max}(N-n)$. Note that the relation between $\hat{\mathbf{R}}_{n|n-1}$ and $\hat{\mathbf{F}}_{N|n-1}$ can be inferred from FIM recursion. The definition in (26) forces the UAV to reach the terminal point $\mathbf{x}_{\mathrm{F}}$ within the total flying time constraint.

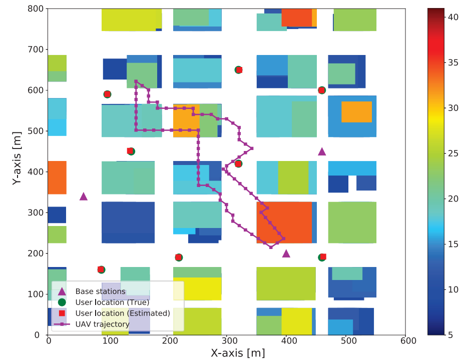
Finally, the next optimal drone position at any time step $n \in [1, N-1]$ can be obtained by solving

$$\begin{aligned} \max_{\mathbf{x}[n+1]} \quad & L(\mathbf{x}[n+1]) \\ \text{s.t.} \quad & \mathbf{x}[1] = \mathbf{x}_{\mathrm{I}}, \\ & \|\mathbf{x}[n+1] - \mathbf{x}[n]\| \leq D_{\max}. \end{aligned} \tag{27}$$

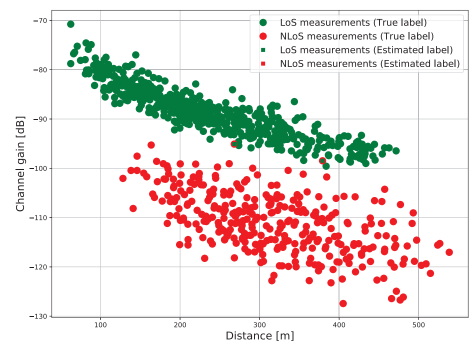
To solve problem (27), we initialize $n = 1$ and the drone starts flying from base point $\mathbf{x}[1] = \mathbf{x}_{\mathrm{I}}$. To find the best drone position in the next time step ($\mathbf{x}[n+1], n \in [1, N-1]$), we discretize the search space around the current drone location and we calculate (26) for all adjacent points. Then the adjacent locations with the maximum value is chosen as the drone location in the next step. In accordance with (24) and (25), the objective $L(\mathbf{x}[n+1])$ is a function of LoS/NLoS labels of measurements which will be collected in the future time step $n+1$. To estimate the labels of the future measurements, we use the LoS probability introduced in [14] which is given by $w_k[n+1] = \frac{1}{1+\exp(a\,\psi[n+1]+b)}$, where $\psi_k[n+1] = \arctan(z[n+1]/r_k[n+1])$ denotes the elevation angle and $r_k[n+1]$ is the ground projected distance between the UAV at time step $n+1$ and the k-th user. Parameters $\{a, b\}$ are the model coefficients which are computed according to [14] and based on the characteristics of the city. Note that if the value of all the neighbor locations are computed as infinity, then the UAV moves towards the terminal location $\mathbf{x}_{\mathrm{F}}$ by $\frac{\|\mathbf{x}[n]-\mathbf{x}_{\mathrm{F}}\|}{N-n}$ meters, where n is the current time step.

V. Numerical Results

A dense urban city neighborhood comprising buildings and streets as shown in Fig. 2-a is considered. The height of the buildings is Rayleigh distributed in the range of 5 to $40\,\mathrm{m}$ [15]. The true propagation parameters are chosen as $\alpha_{\mathrm{LoS}} = -22$, $\alpha_{\mathrm{NLoS}} = -32$, $\beta_{\mathrm{LoS}} = -32\,\mathrm{dB}$, $\beta_{\mathrm{NLoS}} = -35\,\mathrm{dB}$



(a)



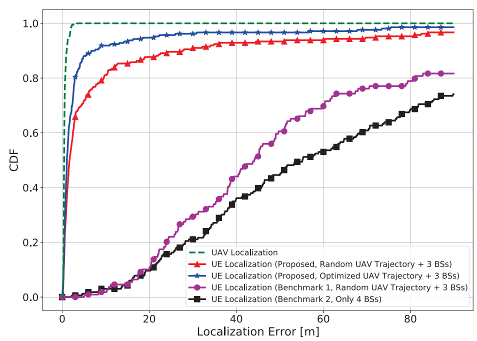
(b)

Fig. 2: (a) The proposed localization algorithm performance for multi-user case and the optimized UAV trajectory, (b) Corresponding channel gain measurements collected from users and classification results. The classification error is 0.2 %.

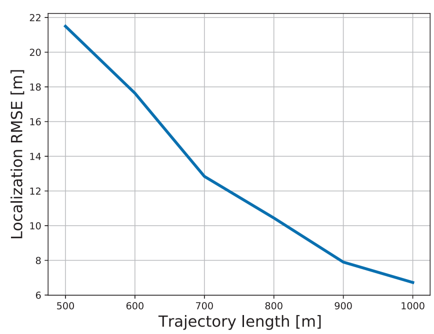
according to an urban micro scenario [16]. The variances of the shadowing components in LoS and NLoS scenarios are $\sigma^2_{\mathrm{LoS}} = 2\,\mathrm{dB}$, and $\sigma^2_{\mathrm{NLoS}} = 5\,\mathrm{dB}$, respectively. For the ToA measurements, the true parameters are chosen as $\mu_{\tau,\mathrm{LoS}} = 0$, $\sigma^2_{\tau,\mathrm{LoS}} = 2$ m, $\mu_{\tau,\mathrm{NLoS}} = 50$ m, $\sigma^2_{\tau,\mathrm{NLoS}} = 40$ m, which is roughly equivalent to 100 MHz bandwidth. We assume that there are three fixed BSs in the city, as shown in Fig. 2-a , with the same altitudes of 25 m. The altitude of the UAV is assumed to be fixed and set to 80 m, and $\mathbf{x}_{\mathrm{I}} = \mathbf{x}_{\mathrm{F}} = [300, 400, 80]^T$. The GPS and the IMU each has a covariance of $\sigma^2_{gps} = 5$ m, $\sigma^2_{vel} = 0.2$ m/s.

In Fig. 2-a, the results of the proposed algorithm for the multi-user case ($K = 8$) are shown. The UAV trajectory is optimized with a maximum length of 1000 m to localize the users. The UAV prioritizes visiting users experiencing the worst conditions (more likely to be NLoS) to enhance localization performance. As a result, the users are accurately localized, with an average localization error of 1.5 m. While Fig. 2-b shows the results of the measurement classification for the same measurement points. For clarity, only the classified channel gain measurements are displayed. The algorithm achieves a low classification error of 0.2 %.

In Fig. 3-a, the cumulative distribution function (CDF) of



(a)



(b)

Fig. 3: (a) The CDF of user localization error for different algorithms., (b) The proposed localization error in terms of RMSE versus increasing the UAV trajectory length.

the user localization error for the proposed algorithm is shown, along with comparisons to different benchmarks over Monte Carlo simulations. The UAV trajectory length is set to 800 m. In Benchmark 1, only RSS measurements are used to localize the users and track the UAV, which can be viewed as an extension of the methods in [13], [17]. Benchmark 2 uses only four static BSs without a UAV. For the random trajectory comparison, the UAV follows a rectangular path of the same length (800 m) instead of the optimized trajectory. It is evident that optimizing the UAV trajectory significantly improves localization performance. Benchmark 2 highlights the performance gain achieved by incorporating mobile anchors. The UAV tracking results (green dashed line) demonstrate that the UAV can be accurately tracked, as it consistently maintains LoS connections with the BSs.

In Fig. 3-b, the performance of the proposed algorithm is shown in terms of root-mean-square error (RMSE) as the UAV trajectory length increases over multiple Monte Carlo simulations. User localization accuracy improves with longer UAV trajectories, as extended trajectory length provides more opportunities to collect LoS measurements, leading to enhanced localization performance.

VI. Conclusion

This paper addresses simultaneous UAV tracking as a mobile anchor and ground user localization. ToA and RSS measurements collected by the UAV are used within an EM-based SLAM framework to classify measurements as LoS/NLoS, learn the radio channel, and jointly localize users and track the UAV. The framework also incorporates GPS-based UAV position estimates and onboard IMU velocity measurements to improve accuracy. Additionally, the UAV trajectory is optimized to collect the most informative measurements, enhancing localization performance.

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