Computation of Rate-Distortion-Perception Function Under f-Divergence Perception Constraints

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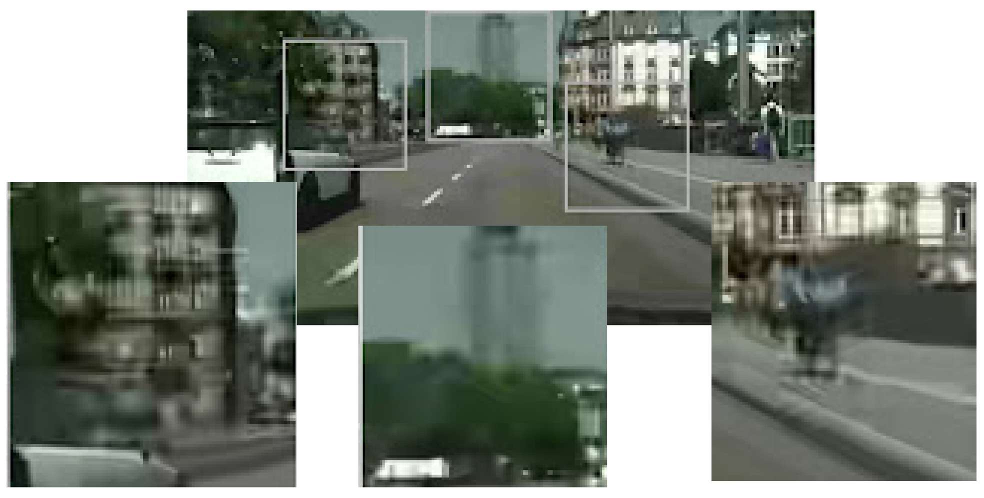
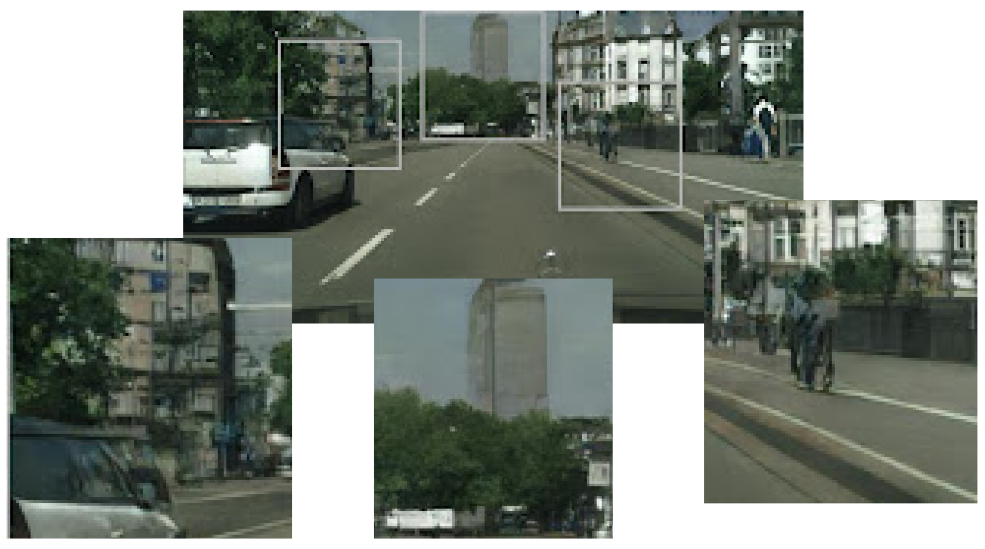
Motivations

In recent years, the application of machine learning techniques in the domains of computer vision and multimedia compression has shown the limits of the traditional source coding framework of rate-distortion (RD) theory. The key insight lies in observing that focusing exclusively on distortion minimization does not imply good perceptual quality, whereas perceptual quality refers to the property of a sample to appear visually pleasing from the human perspective. The introduction of the rate-distortion-perception (RDP) framework generalizes the notion of RD, complementing the classical distortion constraint between a source message and its reconstruction with a divergence constraint between the source distribution and the induced distribution of the reconstructed messages. This additional constraint acts as a proxy of human perception, measuring the degree of satisfaction in the consumption of data from a human perspective.



Bad Perceptual Quality - Low Distortion



Good Perceptual Quality - High Distortion

A different interpretation of this constraint links the divergence measure to a semantic quality metric, which measures the degree of relevance of the reconstructed source from the perspective of the observer.

The Rate-Distortion-Perception (RDP) Function

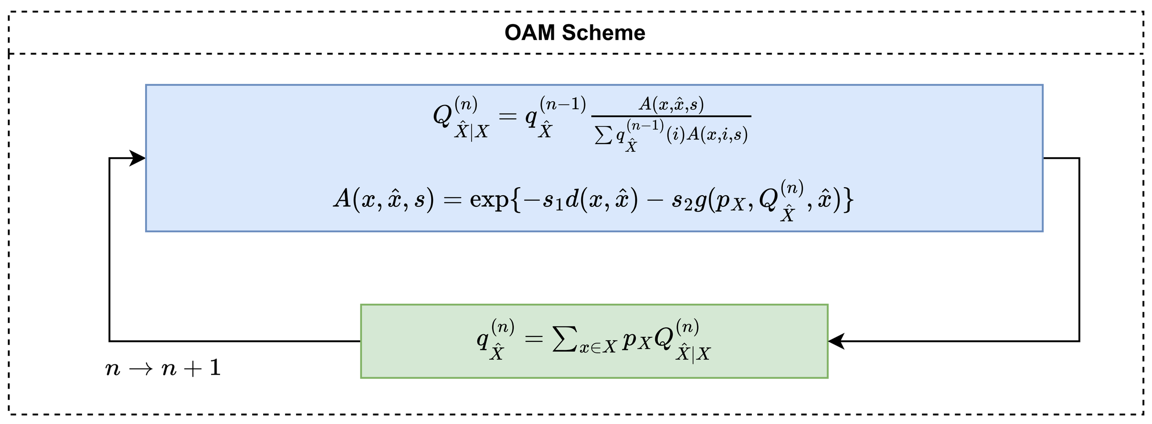
For a given finite alphabet source distribution p_x, a single-letter distortion measure $d(\cdot,\cdot)$ and a f-divergence measure $D_f(\cdot||\cdot)$, the RDPF is characterized as follows:

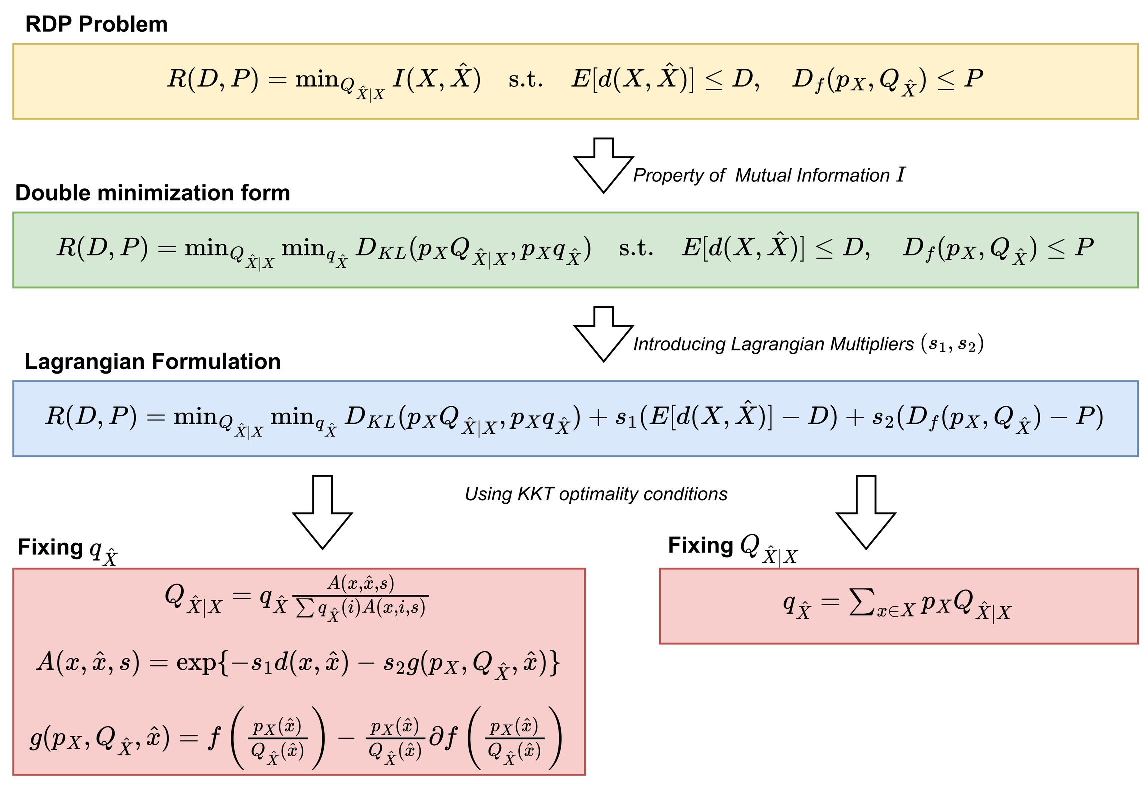
$$R(D,P) = \min_{Q_{\hat{x}|x}} I(X,\hat{X}) \quad \text{s.t.} \quad E\left[d(x,\hat{x})\right] \leq D \qquad D_f(p_x||Q_{\hat{x}}) \leq P$$

where $D \in [D_{\min}, D_{\max}] \subset (0,\infty)$, $P \in [P_{\min}, P_{\max}] \subset (0,\infty)$

Optimal Alternating Minimization (OAM) Scheme

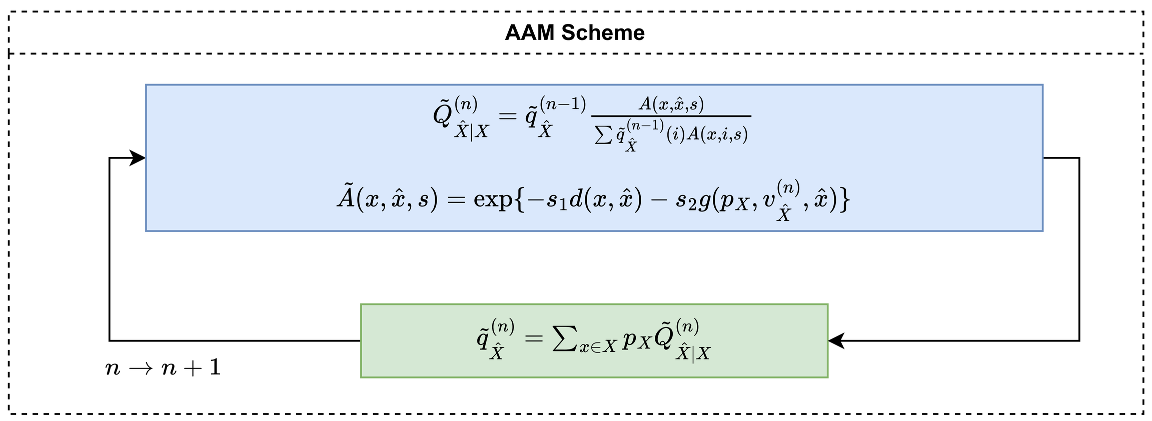
The optimal solutions derived imposing optimality conditions to the RDP problem allow to define an iterative minimization scheme, for which we provide convergence guarantees. However, due to a recursive definition in the iteration structure, the scheme is not easily implementable.

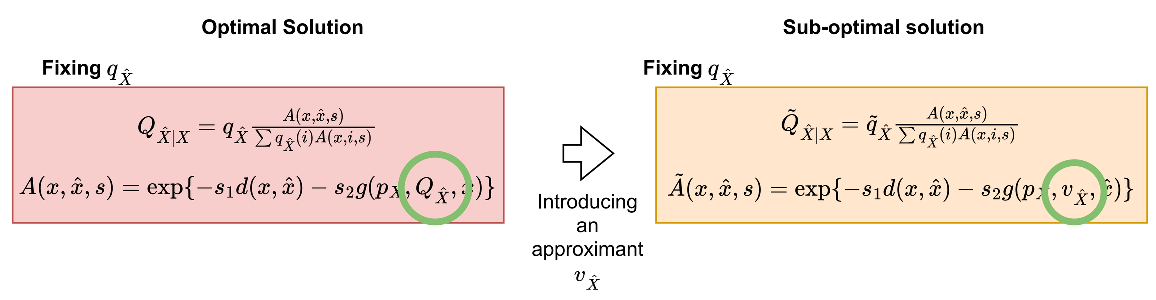




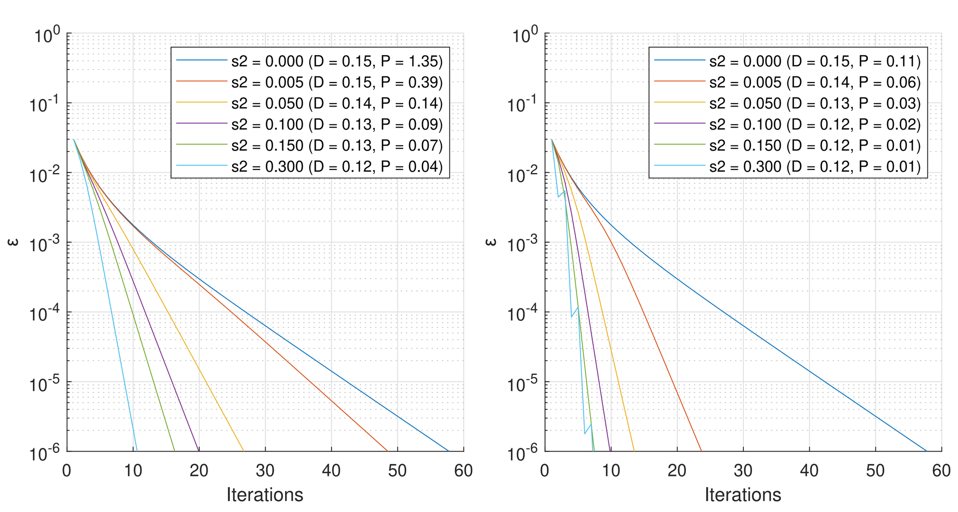
Approximate Alternating Minimization (AAM) Scheme

To circumvent the implementation problem of the OAM scheme, we introduce a relaxation in its iteration structure. Under the condition that the introduced approximation $v^{(n)}$ ensures $\lim_{n\to\infty}(\tilde{q}_{\hat{x}}^{(n+1)} - v_{\hat{x}}^{(n)}) \to 0$ with linear rate of convergence, then the associated AAM scheme converges to the optimal point of the RDP curve.





Asymptotic Convergence Rate Analysis



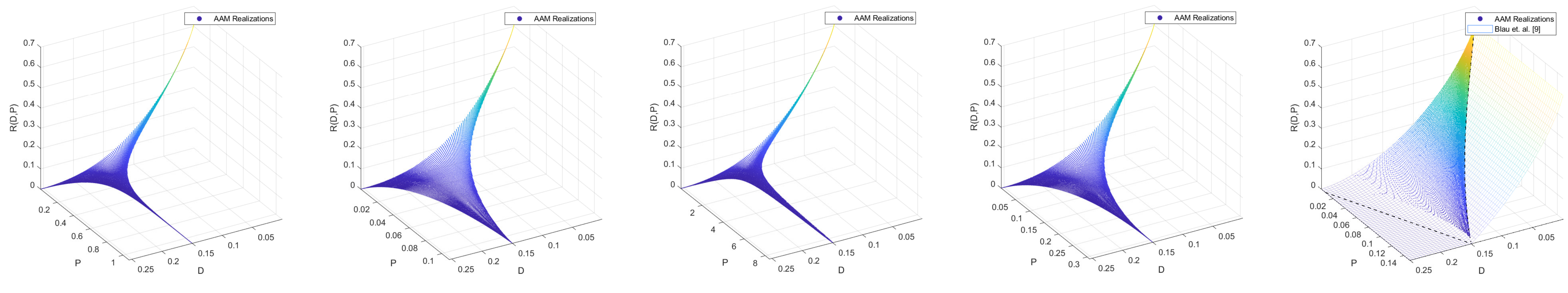
There exist sufficient conditions to ensure the linear convergence of the proposed schemes:

- for the OAM scheme, it suffices that the distortion metric $d: \mathcal{X} \times \hat{\mathcal{X}} \to \mathcal{R}_0^+$ induces a full-rank matrix $K = [\exp\{-s_1 d(i,j)\}]_{(i,j)\in\mathcal{X}\times\hat{\mathcal{X}}}$.
- for the AAM scheme, in addition to the previous condition, we need to constraint the set of allowable Lagrangian multipliers (s_1, s_2).

In the plots we show the convergence behavior of the AAM algorithm for Kullback–Leibler divergence (left) and Jensen-Shannon divergence (right).

Numerical Examples

We consider a binary source alphabet $\{0,1\}$ and a source $X \sim \mathrm{Bern}(0.15)$. The distortion measure is the Hamming distortion $d_H(i,j) = \delta_{i\neq j}$ and the perception measure is selected among commonly used $f-$divergences.



KL divergence | JS divergence | χ^2 divergence | α divergence $(\alpha = \frac{1}{2})$ | TV divergence

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