On Spatio-Frequential Smoothing for joint Angles and Times of Arrival Estimation of Multipaths

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Abstract

A natural extension of the "Spatial" smoothing preprocessing technique is presented and analysed. It is well known that subspace methods do not work properly in the presence of coherent sources. In this paper, a "Spatio-Frequential" smoothing technique is described when the transmit OFDM symbol is received through multiple coherent signals using a uniform linear antenna array. After this preprocessing technique, one could efficiently apply any 2-dimensional subspace method to jointly estimate the angles and times of arrival of the incoming coherent signals. Simulation results demonstrate the potential of the proposed 2D smoothing method over existing separate spatial or frequential smoothing techniques.

Introduction

- The problem of Joint Angle and Delay of arrival Estimation, also known as JADE, is a well-known and challenging problem in the context of array signal processing.
- We focus on the problem of JADE using subspace methods, such as 2D-MUSIC, 2D-ESPRIT, etc.. Subspace algorithms are based on extraction of signal and noise subspaces. These algorithms are computationally much more efficient than ML techniques, but their performance is suboptimal compared to ML.
- In the case of *coherent* sources, i.e. the received signal is a sum of scaled and delayed version of the transmitted signal, all subspace methods fail to estimate angles or times of arrival. Therefore, pre-processing techniques, such as *Spatial* smoothing, have been developed to cope with this issue [1] so as to estimate the angles of arrival using subspace techniques.

Contributions

- We propose a Spatio-Frequential smoothing technique to "decorrelate" the coherent signals so that one could efficiently estimate angles and times of arrival using 2D subspace techniques.

System Model

Consider an OFDM symbol $s(t)$ composed of M subcarriers and centered at a carrier frequency f_c, impinging an antenna array of N antennas via q multipath components, each arriving at different AoAs $\{\theta_i\}_{i=1}^q$ and ToAs $\{\tau_i\}_{i=1}^q$. In baseband, we could write the l^{th} received OFDM symbol at the n^{th} antenna as:

$$\mathbf{x}(l) = \mathbf{A}\boldsymbol{\gamma}(l) + \mathbf{n}(l), \quad l = 1 \dots L \tag{1}$$

where $\mathbf{A} \in \mathbb{C}^{\mathrm{MN}\times q}$ is given by:

$$\mathbf{A} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1) \dots \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)] \quad \text{where} \quad \mathbf{a}(\theta) = [1, z_\theta \dots\ z_\theta^{N-1}]^{\mathrm{T}} \quad \text{and} \quad \mathbf{c}(\tau) = [1, z_\tau \dots\ z_\tau^{M-1}]^{\mathrm{T}} \tag{2}$$

where $z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)}$ and $z_\tau = e^{-j2\pi\triangle_f\tau}$.
The vector $\mathbf{a}(\theta)$ is the ULA array response to a signal arriving at angle θ. Similarly, $\mathbf{c}(\tau)$ is the response of the subcarriers with respect to a signal arriving with delay τ.
The vector $\boldsymbol{\gamma}(l) \in \mathbb{C}^{q\times 1}$ is composed of the multipath coefficients:

$$\boldsymbol{\gamma}(l) = [\gamma_1^{(l)} \dots \gamma_q^{(l)}]^T \tag{3}$$

Assumptions

We assume the following:

- **A1**: $\mathbf{A}$ is full column rank.
- **A2**: The number of multipath components q is known.
- **A3**: The vector $\mathbf{n}(l)$ is additive Gaussian noise of zero mean and covariance $\sigma^2\mathbf{I}_{MN}$, assumed to be white over space, frequencies, and symbols; we also assume that the noise is independent from the multipath coefficients.

Condition **A1** is valid as long as:

- **A1.1**: $q < MN$.
- **A1.2**: We consider that $\forall i \neq j$, $(\theta_i, \tau_i) \neq (\theta_j, \tau_j)$, that is all paths have distinct ToAs and AoAs, simultaneously, but it may happen that two, or more, paths arrive with the same ToAs, but different AoAs.
- **A1.3**: Let q^θ be the number of distinct AoAs, i.e. $\theta^1, \dots, \theta^{q^\theta}$; and let the following integers $Q_1, \dots, Q_{q^\theta}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\theta} Q_i = q$. This condition states that $\max_i Q_i < N$. That is the maximum number of paths arriving at the same time, i.e. $\max_i Q_i$, should be less than N.
- **A1.4**: Similar to **A1.3**, let q^τ be the number of distinct ToAs, i.e. $\tau^1, \dots, \tau^{q^\tau}$; and let the following integers $P_1, \dots, P_{q^\tau}$ denote their corresponding multiplicity. This condition states that $\max_i P_i < M$.
  Note that we have not made any assumptions regarding the multipath vector $\boldsymbol{\gamma}(l)$, therefore we allow coherency of $\boldsymbol{\gamma}(l)$ over l.

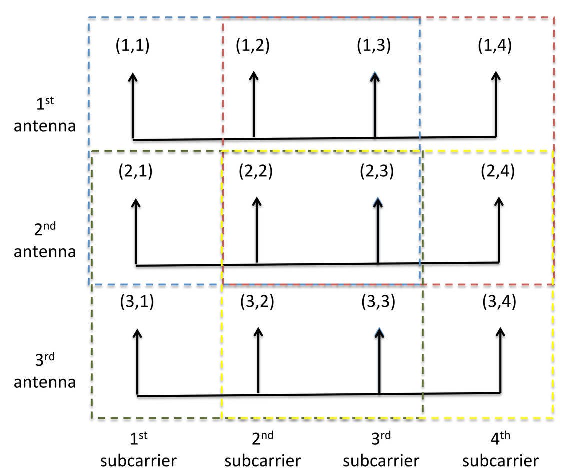
Now, we address our problem:

Given $\{\mathbf{x}(l)\}_{l=1}^L$ and q coherent sources, preprocess the data $\{\mathbf{x}(l)\}_{l=1}^L$ so as to estimate the signal parameters $\{(\theta_i, \tau_i)\}_{i=1}^q$ using a 2D subspace technique.

The Spatio-Frequential Preprocessing Technique

Recall that equation (1) gives the information on all subcarriers at all antennas. We shall use the notation (n, m) to index the m^{th} subcarrier and n^{th} antenna. Let the spatio-frequential array $\{(i,j)\}_{i=1\dots N}^{j=1\dots M}$ of size MN be divided into overlapping subarrays of size M_pN_p (M_p and N_p being the number of subcarriers and antennas in the subarrays, respectively). Indeed, one could check that the total number of overlapping subarrays is equal to K_MK_N, where $K_M = M - M_p + 1$ and $K_N = N - N_p + 1$.

To visualise how the subarrays are formed, we illustrate the figure below, where a setting of $N = 3$ antennas and $M = 4$ subcarriers is partitioned into overlapping subarrays of sizes $N_p = 2$ and $M_p = 3$, and therefore a total of $K_MK_N = 4$ subarrays.



Since the effective number of subcarriers and antennas used now are M_p and N_p, respectively, then (5) becomes

$$\mathbf{x}_{m,n}(l) = \bar{\mathbf{A}}\mathbf{D}_\tau^{m-1}\mathbf{D}_\theta^{n-1}\boldsymbol{\gamma}(l) + \mathbf{n}_{m,n}(l) \quad \text{where} \quad \mathbf{D}_\tau = \mathrm{diag}\,\{z_{\tau_1} \dots z_{\tau_q}\} \quad \text{and} \quad \mathbf{D}_\theta = \mathrm{diag}\,\{z_{\theta_1} \dots z_{\theta_q}\} \tag{4}$$

where $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1) \otimes \bar{\mathbf{c}}(\tau_1) \dots \bar{\mathbf{a}}(\theta_q) \otimes \bar{\mathbf{c}}(\tau_q)]$ is an $M_pN_p \times q$ matrix. The vectors $\mathbf{a}(\theta)$ and $\mathbf{c}(\tau)$ are the same as in (2) but with sizes N_p and M_p instead of N and M, respectively. $\mathbf{D}_\tau^{m-1}$ and $\mathbf{D}_\theta^{n-1}$ are the $(m-1)^{th}$ and $(n-1)^{th}$ power of the diagonal $q \times q$ matrices $\mathbf{D}_\tau$ and $\mathbf{D}_\theta$. This means that $\mathbf{x}_{m,n}(l)$ is an $M_pN_p \times 1$ received vector on the subarray $\{(i,j)\}_{j=n\dots N_p+n-1}^{i=m\dots M_p+m-1}$. The covariance matrix of $\mathbf{x}_{m,n}(l)$ in (4) after averaging over time snapshots is given as

$$\mathbf{R}_{m,n} = \bar{\mathbf{A}}\mathbf{D}_\tau^{m-1}\mathbf{D}_\theta^{n-1}\mathbf{R}_{\gamma\gamma}\mathbf{D}_\theta^{\mathrm{H}^{n-1}}\mathbf{D}_\tau^{\mathrm{H}^{m-1}}\bar{\mathbf{A}}^{\mathrm{H}} + \sigma^2\mathbf{I}_{M_pN_p} \tag{5}$$

The *spatio-frequential smoothed covariance* matrix is given by

$$\bar{\mathbf{R}} = \frac{1}{K_MK_N}\sum_{m=1}^{K_M}\sum_{n=1}^{K_N}\mathbf{R}_{m,n} = \bar{\mathbf{A}}\bar{\mathbf{R}}_{\gamma\gamma}\bar{\mathbf{A}}^{\mathrm{H}} + \sigma^2\mathbf{I}_{M_pN_p} \quad \text{where} \quad \bar{\mathbf{R}}_{\gamma\gamma} = \frac{1}{K_MK_N}\sum_{m=1}^{K_M}\sum_{n=1}^{K_N}\mathbf{D}_\tau^{m-1}\mathbf{D}_\theta^{n-1}\mathbf{R}_{\gamma\gamma}\mathbf{D}_\theta^{\mathrm{H}^{n-1}}\mathbf{D}_\tau^{\mathrm{H}^{m-1}} \tag{6}$$

In a single carrier case, i.e. $M = M_p = 1$, it has been proven that the spatial smoothing technique ensures full rank of $\bar{\mathbf{R}}_{\gamma\gamma}$ [1], given that $q \leq K_N$.
Analogously, in the single antenna but multi-carrier case, i.e. $N = N_p = 1$, the same technique has been applied in [2] and was referred to as frequency smoothing, in order to acheive full rank of $\bar{\mathbf{R}}_{\gamma\gamma}$, when $q \leq K_M$. However, in the general multi-antenna and multi-carrier case, we have the following:

***Theorem*: If the following hold true:**

- ***The number of subarrays formed jointly over space and frequency is greater than the number of multipath components, i.e. $q \leq K_MK_N$.***
- ***The maximum number of paths arriving at the same time but with different angles is less than K_N, i.e. $\max_i Q_i \leq K_N$.***
- ***The maximum number of paths arriving at the same angles but with different times is less than K_M, i.e. $\max_i P_i \leq K_M$.***

Then $\bar{\mathbf{R}}_{\gamma\gamma}$ is of rank q.

***Proof*: See our paper.**

Finally, the advantage of *spatio-frequential* smoothing is that it offers K_MK_N subarrays to smooth over, in contrast to *spatial* and *frequential* smoothing that naturally provide K_N and K_M subarrays, respectively. Therefore, one could be able to resolve more coherent sources and provide better ToA/AoA estimates.

Simulations

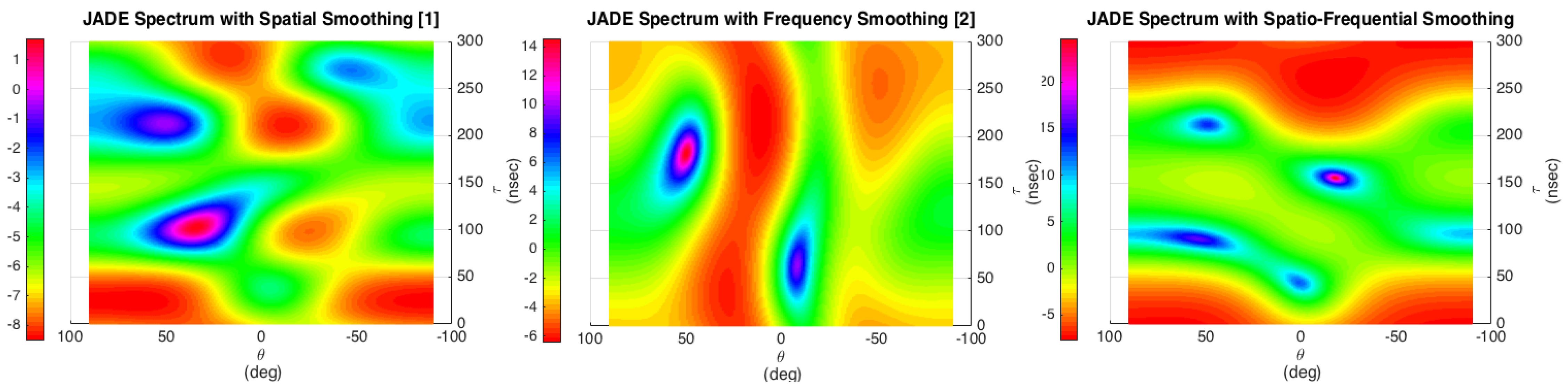


Figure 1: JADE-MUSIC Spectrum of a scenario where $q = 4$ multipath components are present. Simulations have been done with $N = 3$ antennas and $M = 4$ subcarriers at SNR = 20dB. The subcarrier spacing is chosen $\triangle_f = 3.125$ MHz. The complex attenuation vector γ is fixed to a constant arbitrary value. Finally, $L = 3$ snapshots were collected.

Figure 1(a) shows the JADE spectrum after preprocessing only by spatial smoothing, i.e. $M = M_p = 4$ and $N_p = 2$. Indeed, there is an ambiguity in detecting the 4 peaks corresponding to the true angles and times of arrival due to insufficient number of subarrays to smooth over, i.e. only $K_N = 2 < q$ spatial subarrays are available. The same argument is done when one applies only frequency smoothing, i.e. $N = N_p = 3$ and $M_p = 2$. In that case, one will have $K_M = 3 < q$ subarrays to smooth over. As a result, false peaks appear in Figure 1(b). To this end, we could see that we need at least $q = 4$ subarrays to smooth over. This is done by preprocessing through spatio-frequential smoothing. Choosing $N_p = 2$ and $M_p = 3$ would lead to $K_NK_M = 4$ subarrays in total. After smoothing over space and frequencies, one could observe 4 clear peaks corresponding to the true angles and times of arrival of the 4 paths in Figure 1(c).

Conclusions

- We have presented a 2D smoothing preprocessing technique, applied to a *Spatial-Frequential* array, to "decorrelate" multipath components.
- Then, any 2D subspace algorithm could be applied to estimate the times and angles of arrivals of the different paths.
- The 2D smoothing technique presented here, naturally, offers more subarrays to smooth over and, therefore, one could be able to resolve more coherent paths.

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