Equilibriums in Slow Fading Interfering Channels with Partial Knowledge of the Channels

Lei Xiao
Eurecom
Sophia Antipolis, France
lei.xiao@eurecom.fr

Laura Cottatellucci
Eurecom
Sophia Antipolis, France
cottatel@eurecom.fr

Konstantin Avrachenkov
INRIA
Sophia Antipolis, France
kavratch@sophia.inria.fr

***Abstract*—We consider a block fading interference channels with partial channel state information and we address the issue of joint power and rate allocation in a game theoretic framework. The system is intrinsically affected by outage events. Resource allocation algorithms based on Bayesian games are proposed. The existence, uniqueness, and some stability properties of Nash equilibriums (NE) are analyzed. For some asymptotic setting, closed form expressions of NEs are also provided.**

I. Introduction

The large gain in spectral efficiency achievable by sharing the complete frequency spectrum is fueling intense research activities on the interference channel. The interference channel is intrinsically characterized by a limited level of cooperation among communication entities, which are rather competing for the same resources, and by a decentralized resource management. These complex interactions can be modeled successfully in a game theoretical framework. This direction of investigation is currently receiving considerable attention (see e.g. [1], [2], [3], [4], [5]). Many contributions focus on the channels with complete channel state information at the transmitters. Alternatively, iterative algorithms are proposed whose convergence to an equilibrium point is based on feedbacks from the receivers. A well known and thoroughly studied example of this class of algorithms is the iterative waterfilling algorithm suitable for frequency selective interference channels (see [3] and references therein). The convergence speed of these algorithms limits their applicability. Additionally, the required feedbacks reduce the system spectral efficiency. In [2], slow fading channels are considered with slow fading and initial partial channel state information. By using the approach of repeated games, information about the channel and the interactions is acquired. When the constraints of a communication system do not allow for the convergence of iterative algorithms (e.g. systems whose channels can be considered constant during the transmission of a codeword with constrained length but still varying from codeword to codeword or channels with constrained delay capacities described in [6]) or do not support the intensive feedbacks required by iterative algorithms, Bayesian games provide a convenient theoretical framework. Resource allocation based on Bayesian games are adopted in [7], [8], [9]. The works in [7], [8] focus on fast fading channels while the Bayesian game in [9] is applied to slow fading multiple access channels based on orthogonal frequency division multiplexing (OFDM). Interestingly, [9] shows that the throughput achievable via the resource allocation based on Bayesian games has performance comparable to a resource allocation based on an optimization that assumes full channel state information at the transmitter.

In this work we consider a block fading interference channel with knowledge of the state of the direct links but only statistical knowledge on the interfering links. With this assumption, reliable communications are not possible and a certain level of outage has to be tolerated. We consider the resource allocation for utility functions based on the real throughput accounting for the outage events. In the extended technical report [10] we propose resource allocation algorithms based on both *Bayesian games* and *optimization*. In the context of Bayesian games, we investigate the two cases of *power allocation for predefined transmission rates* and *joint power and rate allocation*. In this paper, due to space constraints, we focus on a joint power and rate allocation algorithm based on a Bayesian game. For the same reason, we omit here both the analytical proofs of the theoretical results and the numerical analysis. We refer the interested reader to [10]. In order to provide a deeper insight on the behaviour of the analyzed system, we briefly recapitulate also the most relevant features of both the optimum resource allocation and the power allocation for predefined transmission rate based on Bayesian games in this introduction.

The Bayesian power allocation for predefined transmission rates boils down to a concave game. Thus, NEs always exist [11] and they are at most three [10]. Some sufficient conditions for the uniqueness of the NE are also available [10]. On the contrary, the Bayesian game for joint power and rate allocation is not concave and its analysis is based on the analysis of an equivalent game. The characteristics of the game theoretical approaches are analyzed in terms of existence, multiplicity, and stability of the NEs. Special attention is devoted to the asymptotic high noise regime and the interference limited regime. In the former case, a closed form expression for the Nash equilibrium is provided. In the latter case, criteria for the convergence of best response algorithms are discussed. In [10], the optimization approach is also analyzed in the two above mentioned regimes and closed form expressions for the resource allocation are provided. Interestingly, in the asymptotic regimes the optimum allocation implies a condition of starvation for one communication while the resource allocation based on Bayesian games is fairer [10].

II. System Model

Let us consider an interference channel with two sources $\mathcal{S}_1, \mathcal{S}_2$ and two destinations $\mathcal{D}_1, \mathcal{D}_2$. The two sources transmit independent information and source $\mathcal{S}_i$ aims at communicating with destination $\mathcal{D}_i$, for $i = 1, 2$. We assume that the channel is block fading, i.e. the channel gains of all the links are constant in the timeframe of a codeword but are independent and identically distributed from codeword to codeword. Note that these channels are often referred to as quasistatic channels or as channels with delay-limited capacity [6]. We denote by g_i, $i = 1, 2$, the channel power gains of the direct links $\mathcal{S}_1 - \mathcal{D}_1$ and $\mathcal{S}_2 - \mathcal{D}_2$ and by h_{12} and h_{21} the channel power gains of the interfering links $\mathcal{S}_1 - \mathcal{D}_2$ and $\mathcal{S}_2 - \mathcal{D}_1$. All the channel gains fade independently such that the channel power gain statistics are completely determined by the marginal distributions. Each source transmits only private information that can be decoded only by its targeted destination, or equivalently, each receiver performs single user decoding. Additionally, each source knows the realizations of both direct links g_1 and g_2 but not the realizations of the power gains h_{12} and h_{21} for the interfering links. This corresponds to a typical situation (e.g. in cellular systems) where the receivers estimate only the channel gains of the direct links and feed them back to the transmitter but neglect the interfering links. Throughout this work we make the additional assumption that the power gains of the interfering links are Rayleigh distributed, i.e. their probability density function is given by $\gamma_{H_{ij}}(h_{ij}) = \frac{1}{\sigma_{ij}^2} \mathrm{e}^{-\frac{h_{ij}}{\sigma_{ij}^2}}$. Furthermore, these statistics are known to both sources. At the receiver the channel is impaired by additive Gaussian noise with variance N_0.

III. Problem Statement

Because of the partial knowledge of the channel by the sources and the assumption of block fading, reliable communications, i.e. with error probability arbitrarily small, are not feasible (e.g. [12]) and outage events may happen. If the source i transmits at a certain rate, expressed in nat/sec, with constant transmitted power P_i, an outage event happens if

$$R_i > \log\left(1 + \frac{P_i g_i}{N_0 + P_j h_{ji}}\right), \quad i,j = 1,2,\ i \neq j, \tag{1}$$

and the outage probability of source i depends on the choice of R_i, P_i and P_j. We define the throughput as the average information that can be correctly received by the destination. The throughput is given by

$$T_i(P_i, R_i, P_j) = R_i \Pr\left\{R_i \leq \log\left(1 + \frac{P_i g_i}{N_0 + P_j h_{ji}}\right)\right\} \tag{2}$$

where $i, j = 1, 2$ with $i \neq j$, and $\Pr\{\mathcal{E}\}$ denotes the probability of the event $\mathcal{E}$.

The two sources need to determine autonomously and in a decentralized manner the transmitting power P_i and the rate R_i. A natural criterion is to allocate such resources in order to maximize the throughput while keeping power consumption moderate. Then, we define the objective function for source $\mathcal{S}_i$ as

$$u_i((P_i, R_i), (P_j, R_j)) = T_i(P_i, R_i, P_j) - C_i P_i \tag{3}$$

where C_i is the cost for unit power.

By making use of the assumption on the power gain distributions of the interfering links, the utility of $\mathcal{S}_i$ is given by

$$\begin{aligned} & u_i((R_i, P_i), (R_j, P_j)) \\ & = R_i \Pr\left\{R_i \leq \log\left(1 + \frac{P_i g_i}{N_0 + P_j h_{ji}}\right)\right\} - C_i P_i \\ & = \begin{cases} R_i F_i(t_i) - C_i P_i, & \{P_j > 0,\ P_i, R_i \geq 0\} \setminus \{P_i = R_i = 0\}; \\ 0, & \{P_j > 0,\ P_i = R_i = 0\}; \\ R_i - C_i P_i, & \{P_j = 0, R_i, P_i \geq 0, P_i \geq \frac{(\mathrm{e}^{R_i} - 1) N_0}{g_i}\}; \\ -C_i P_i, & \{P_j = 0, R_i, P_i \geq 0, P_i \leq \frac{(\mathrm{e}^{R_i} - 1) N_0}{g_i}\}; \end{cases} \end{aligned} \tag{4}$$

where $t_i = \frac{P_i g_i}{\mathrm{e}^{R_i} - 1} - N_0$, $F_i(t_i) = 1 - \exp\left(-\frac{t_i}{P_j \sigma_{ij}^2}\right)$ and C_i is the cost of unit power by user i.

Since the objective function of $\mathcal{S}_i$ depends also on the power allocated by $\mathcal{S}_j$ the problem falls naturally in the framework of strategic games. Then, the objective of source $\mathcal{S}_i$ is to determine the pair (P_i, R_i), that selfishly maximizes its utility function $u_i((P_i, R_i), (P_j, R_j))$ under the assumption that a similar strategy is adopted by the other source.

IV. Interference Games for Joint Power and Rate Allocation

In this section we consider a communication system where the transmitters need to allocate both power and rate jointly with the aim of maximizing the utility function (3). The problem is defined as a strategic game $\mathcal{G} = \{\mathcal{S}, \mathcal{P}, \{u_i\}_{i \in \{1,2\}}\}$, where $\mathcal{S}$ is the set of players (the two transmitters), $\mathcal{P}$ is the strategy set defined by $\mathcal{P} \equiv \{((P_1, R_1), (P_2, R_2)) | P_1, P_2, R_1, R_2 \geq 0\}$, and u_i is the utility function defined in (3). Power and rate allocation is obtained as an equilibrium point of the system. When both transmitters aim at maximizing their utility functions, a NE is the allocation strategy $(P_1^*, R_1^*, P_2^*, R_2^*)$ such that

$$\begin{aligned} u_1(P_1^*, R_1^*, P_2^*, R_2^*) &\geq u_1(P_1, R_1, P_2^*, R_2^*),\ \forall P_1, R_1 \in \mathbb{R}_+ \\ u_2(P_1^*, R_1^*, P_2^*, R_2^*) &\geq u_2(P_1^*, R_1^*, P_2, R_2),\ \forall P_2, R_2 \in \mathbb{R}_+. \end{aligned}$$

It is straightforward to verify that the utility function is not concave in R_i. Then, the classical results on N-concave games in [11] cannot be applied. The analysis of the general case results very complex. A preliminary characterization of NEs for game $\mathcal{G}$ is provided in the following proposition. This proposition provides closed form expressions for the NEs at the boundary of the strategy set jointly with explicit conditions for the points being NEs. Possible NEs internal to the strategy set are provided in an implicit form and they will be further analyzed in subsequent propositions.

Proposition 1 *A boundary point of the strategy set $\mathcal{P}$ is a NE if and only if*

$$P_i = R_i = 0 \quad (5)$$

$$P_j = \frac{1}{C_i} - \frac{N_0}{g_j} \qquad R_j = \log\left(1 + \frac{g_j - N_0 C_j}{N_0 C_j}\right) \quad (6)$$

and the following conditions are satisfied

$$g_j - N_0 C_j \geq 0 \quad (7)$$

$$\frac{g_i \alpha_j}{C_i \sigma_{ji}^2} \exp\left(-\frac{g_i \alpha_j}{C_i \sigma_{ji}^2} + \frac{1}{N_0 \alpha_j} + 1\right) \geq 1, \quad (8)$$

being $\alpha_j = \frac{C_j g_j}{g_j - N_0 C_j}$.

An internal point of the strategy set $\mathcal{P}$ is a NE if and only if it is solution of the system of equations

$$\frac{1}{P_j \sigma_{ji}^2} \exp\left(-\frac{t_i}{P_j \sigma_{ji}^2}\right) = \frac{C_i(\mathrm{e}^{R_i} - 1)}{R_i g_i} \qquad i,j = 1,2. \quad (9)$$

where $t_i = \frac{P_i g_i}{\mathrm{e}^{R_i}-1} - N_0$ *and P_1 and P_2 are given as functions of R_1 and R_2 by*

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} C_1 \frac{\mathrm{e}^{R_1}}{\mathrm{e}^{R_1}-1} & \frac{C_1 \sigma_{21}^2}{R_1 g_1}(\mathrm{e}^{R_1} - 1) \\ \frac{C_2 \sigma_{12}^2}{R_2 g_2}(\mathrm{e}^{R_2} - 1) & C_2 \frac{\mathrm{e}^{R_2}}{\mathrm{e}^{R_2}-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (10)$$

and it satisfies the following inequalities

$$1 + R_i + \frac{g_i R_i}{C_i P_j \sigma_{ji}^2 (\mathrm{e}^{R_i} - 1)} - \frac{2 R_i \mathrm{e}^{R_i}}{\mathrm{e}^{R_i} - 1} > 0 \quad (11)$$

$$\frac{R_i^2 g_i}{C_i P_j \sigma_{ji}^2 (\mathrm{e}^{R_i} - 1)} - R_i - \left(1 - \frac{R_i \mathrm{e}^{R_i}}{\mathrm{e}^{R_i} - 1}\right)^2 > 0. \quad (12)$$

In order to get additional insights into the system behavior and in particular into the NEs internal to the strategy set $\mathcal{P}$, we consider firstly the following extreme cases before discussing the general case: (1) the noise tends to zero, (*interference limited regime*), (2) the noise is much higher than the transmitted power (*high noise regime*).

a) Interference Limited Regime: When the noise variance N_0 is negligible compared to the interference power level, the payoff function is efficiently approximated by (4), with $t_i = \frac{P_i g_i}{\mathrm{e}^{R_i}-1}$. Note that in the interference limited regime, the payoff (4) of user i is defined for $0 \leq N_0 \ll P_j$. In the following proposition equilibriums of game $\mathcal{G}$ are obtained as equilibriums of an equivalent game in a single decision variable x_i for user i.

Proposition 2 *When the noise variance tends to zero, the NE of game $\mathcal{G}$ and internal to $\mathcal{P}$ satisfy the system of equations*

$$x_1 = \kappa_2 f(x_2) \quad (13)$$
$$x_2 = \kappa_1 f(x_1)$$

where $x_i = \frac{g_i}{C_i P_j \sigma_{ji}^2}$, $\kappa_i = \frac{C_i g_j}{C_j \sigma_{ij}^2}$, $i, j \in 1, 2$, $i \neq j$ *and*

$$f(x) = \left(1 - \frac{\mathrm{e}^{R(x)} - 1}{x R(x)}\right)^{-1} \left(1 - \mathrm{e}^{-R(x)}\right)^{-1} \quad (14)$$

for $1 < x < \infty$. In (14), $R(x)$ is the unique ***positive*** *solution of the equation*

$$1 - \frac{xR}{\mathrm{e}^R - 1} \exp\left(-\frac{x}{\mathrm{e}^R} + \frac{\mathrm{e}^R - 1}{R\mathrm{e}^R}\right) = 0 \quad (15)$$

such that

$$-x + \frac{\mathrm{e}^R - 1}{R} \neq 0. \quad (16)$$

Let (x_1^0, x_2^0) be solutions of system (13). The corresponding NE is given by

$$P_1 = \frac{g_2}{C_2 x_2^0 \sigma_{12}^2}, \quad R_1 = R(x_1^0),$$
$$P_2 = \frac{g_1}{C_1 x_1^0 \sigma_{21}^2}, \quad R_2 = R(x_2^0).$$

Remarks

- The solution $\overline{R}(x)$ to the equation $\frac{\mathrm{e}^R - 1}{R} = x$ is also a solution to (15). Such a solution corresponds to a minimizer of the utility function.
- The solution $R(x_j)$ to (15) is the rate which maximizes the utility function corresponding to the transmit power of the other transmitter $P_i = \frac{g_j}{C_j x_j \sigma_{ij}^2}$. It lies in the interval $\left(0, \overline{R}(x_j)\right)$ and we refer to it as the *best response in terms of rate* of player j to strategy P_i of player i. Similarly, $\kappa_j f(x_j)$ is inverse proportional to the *best response in terms of power* of user j to the strategy P_i of its opponent.
- Interestingly, the solution (x_1^0, x_2^0) to system (13) depends on the system parameters only through the constants κ_1 and κ_2.
- The existence and uniqueness of NE for the class of systems considered in Proposition 2 reduces to the analysis of the solution of system (13) and depends on the system via κ_1 and κ_2.
- The solution to (15) can be effectively approximated by $R(x) \approx 0.8 \log(x)$. Then, the function $f(x)$ is approximated by

$$\widetilde{f}(x) = \left(1 - \frac{\mathrm{e}^{0.8 \log(x)} - 1}{x 0.8 \log(x)}\right)^{-1} \left(1 - \mathrm{e}^{-0.8 \log(x)}\right)^{-1}. \quad (17)$$

In Figure 2, $f(x_i)$ is plotted and compared to its approximation $\widetilde{f}(x)$. The approximation $\widetilde{f}(x)$ matches almost perfectly $f(x)$ such that can be utilized efficiently for practical and analytical objectives.

The following proposition provides sufficient conditions for the existence of a NE.

Proposition 3 *When the noise variance is negligible compared to the interference, a NE of the game $\mathcal{G}$ exists if*

$$(\kappa_1 - 1)(\kappa_2 - 1) > 0$$

with κ_i defined in Proposition 2.

General conditions for the uniqueness of a NE are difficult to determine analytically. Let us observe that in general a system with noise that tends to zero may have more than

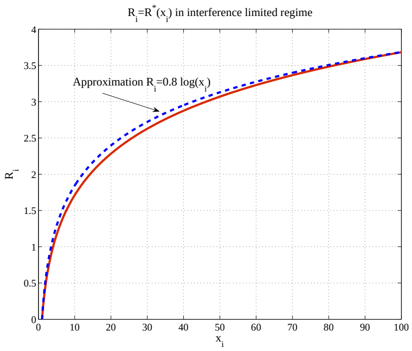

Fig. 1: Best response $R^*(x_i)$ of user i to the transmitted power $P_j = \frac{g_i}{x_i \sigma_{ji} C_i}$ in solid line and its approximation $0.8 \log x_i$ in dashed line.

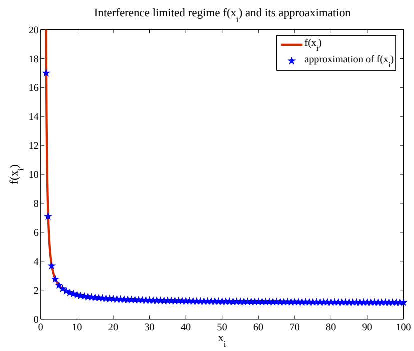

Fig. 2: $f(x_i)$ in solid line and its approximation $\widetilde{f}(x_i)$

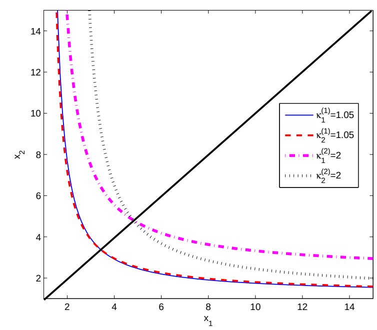

Fig. 3: Graphical investigation of convergence of the best response algorithm in the interference limited regime

one NE. Let us consider a system with $\kappa_1 = \kappa_2$. The two curves $x_j = \kappa_i f(x_i)$, $i, j \in \{1, 2\}, i \neq j$, cross each other in $x_1 = x_2$. Furthermore, the curve $x_2 = \kappa_1 f(x_1)$ ($x_1 = \kappa_2 f(x_2)$) has two asymptotes in $x_1 = 1$ and $x_2 = \kappa_1$ ($x_2 = 1$ and $x_1 = \kappa_2$). Then, for $\kappa_1 = \kappa_2 = 1$, the two curves cross again in $(1, +\infty)$ and $(+\infty, 1)$. Let us observe now Figure 3 where the best responses of the two systems corresponding to the two pairs of coefficients $\kappa_1^{(1)} = \kappa_2^{(1)} = 1.05$ and $\kappa_1^{(2)} = \kappa_2^{(2)} = 2$ are plotted. It becomes apparent that the curves with $\kappa_1^{(1)} = \kappa_2^{(1)} = 1.05$ will cross again for high x_1 and x_2 values since $x_2 = \kappa_1^{(1)} f(x_1)$ has two asymptotes in $x_1 = 1$ and $x_2 = 1.05$ while $x_1 = \kappa_2^{(1)} f(x_2)$ has two asymptotes in $x_2 = 1$ and $x_1 = 1.05$. These crossing points correspond to NEs. In contrast, the curves with $\kappa_1^{(1)} = \kappa_2^{(1)} = 2$ will diverge from each other. It is worth noticing that for $x_1 \gg 1$, $x_2 \approx 1$, and for $x_2 \gg 1$, $x_1 \approx 1$. From a telecommunication point of view, it is necessary to question whether the model for $N_0 \ll P_j g_j$ is still applicable. In fact, in such a case, $P_i \ll \frac{g_i}{C_i \sigma_{ij}^2}$, but also $P_i \gg N_0$ has to be satisfied because of the system model assumptions. Typically, the additional NEs with some $x_i \approx 1$ are not interesting from a physical point of view since the system model assumptions are not satisfied. Thus, additional NEs are artifacts introduced by the asymptotic model.

By numerical simulations, we could observe that games with multiple NEs exist for a very restricted range of system parameters, more specifically for $1 \leq \kappa_i \leq 1.1$.

Proposition 2 suggests also an iterative algorithm for computing NE based on the best response. Choose an arbitrary point $x_1^{(0)}$ and compute the corresponding value $x_2^{(0)} = \kappa_1 f(x_1^{(0)})$. From a practical point of view, this is equivalent to choose arbitrarily the transmitted power $P_2^{(0)} = \frac{g_1}{\sigma_{21}^2 x_1^{(0)} C_1}$ for transmitter 2 and determine the power allocation for user 1 which maximizes its utility function. The optimum power allocation for user 1 is $P_1^{(0)} = \frac{g_2}{\sigma_{12}^2 x_2^{(0)} C_2}$. We shortly refer to $P_1^{(0)}$ as the best response of user 1 to user 2. Then, by using $x_2^{(0)}$ it is possible to compute $x_1^{(1)} = \kappa_2 f\left(x_2^{(0)}\right)$, the best response of user 2 to user 1. By iterating on the computation of the best responses of user 1 and user 2 we can obtain resource allocations closer and closer to the NE and converge to it. We refer to this algorithm as the best response algorithm.

The best response algorithm is very appealing for its simplicity. Nevertheless, its convergence is not guaranteed. This issue is illustrated in Figure 3. Let us consider the interference channel with $\kappa_1 = \kappa_2 = 1.05$ and the corresponding solid and dashed curves $x_2 = \kappa_1 f(x_1)$ and $x_1 = \kappa_2 f(x_2)$. The NE exists and is unique but the best response algorithm diverges from the NE even for choices of the initial point arbitrarily close to the NE but different from it. Numerical results show that if κ_1 and κ_2 are both greater than 1.1, the best response algorithm always converges to a NE.

The following analytical result holds.

Proposition 4 *For sufficiently large κ_1 and κ_2, the fixed point iterations*

$$\begin{cases} x_1^{(k+1)} = \kappa_2 f(x_2^{(k)}), \\ x_2^{(k+1)} = \kappa_1 f(x_1^{(k)}), \end{cases} \tag{18}$$

converge.

In fact, large values of κ_1 and κ_2 correspond to a realistic situation for system where the noise is negligible compared to the transmitted powers of the users.

A. High Noise Regime

Let us turn to the case when noise is much higher than the useful received power, $P_i g_i \ll N_0$. The throughput can be approximated by

$$\begin{aligned} \overline{T}_i\left(P_i, R_i, P_j, P_*\right) &= R_i \Pr\left\{R_i \leq \frac{P_i g_i}{N_0 + P_j h_{ji}}\right\} \\ &= R_i \Pr\left\{h_{ji} \leq \frac{1}{P_j}\left(P_i \frac{g_i}{R_i} - N_0\right)\right\} \end{aligned} \tag{19}$$

Interestingly, the throughput in (19) is nonzero for $\frac{P_k}{R_k} > \frac{N_0}{g_k}$. Since Proposition 1 defines completely the NEs on the boundary of the strategy set in the general case, in this section we focus only on internal points of $\mathcal{P}$. Then, the utility function is given by

$$v_i = R_i \left(1 - \exp\left(-\frac{\left(P_i \frac{g_i}{R_i} - N_0\right)}{P_j \sigma_{ji}^2}\right)\right) - C_i P_i \qquad (20)$$

for $i = 1, 2$. Correspondingly, we consider the game $\overline{\mathcal{G}} = \left\{\mathcal{S}, \mathcal{V}, \overset{o}{\mathcal{P}}\right\}$, where the set of players coincides with the corresponding set in $\mathcal{G}$ while the utility function set $\mathcal{V}$ consists of the functions (20) and $\overset{o}{\mathcal{P}}$ is the open interval obtained from $\mathcal{P}$. The joint rate and power allocation is given by NE of game $\overline{\mathcal{G}}$.

The following proposition states the conditions for the existence and uniqueness of a NE in the strategy set and provides the equilibrium point.

Proposition 5 *Game $\overline{\mathcal{G}}$ admits a NE if and only if*

$$\frac{g_i}{C_i} > N_0, \quad i = 1, 2.$$

If the above conditions are satisfied, $\overline{\mathcal{G}}$ has a unique equilibrium $\left((R_i^*, P_i^*), (R_j^*, P_j^*)\right)$ where P_i^* and P_j^* are the unique roots of the equations

$$\left(1 - \ln\left(\frac{C_j P_i \sigma_{ij}^2}{g_j}\right)\right) P_i \sigma_{ij}^2 = \frac{g_j}{C_j} - N_0 \qquad (21)$$

and

$$\left(1 - \ln\left(\frac{C_i P_j \sigma_{ji}^2}{g_i}\right)\right) P_j \sigma_{ji}^2 = \frac{g_i}{C_i} - N_0 \qquad (22)$$

in the intervals $\left(0, \frac{g_j}{C_j \sigma_{ij}^2}\right)$ and $(0, \frac{g_i}{C_i \sigma_{ji}^2})$ respectively. Also,

$$R_i = \frac{P_i g_i C_i}{g_i - P_j \sigma_{ji}^2 C_i} \quad \text{and} \quad R_j = \frac{P_j g_j C_j}{g_j - P_i \sigma_{ij}^2 C_j}.$$

Interestingly, the power allocation of user i decouples from the one of user j and P_i depends on its opponent only via the system parameter ratio $\frac{C_j}{g_j}$.

B. General Case

Let us consider now the general case, when the noise, the powers of interferences and the transmitted powers are of the same order of magnitude. A NE necessarily satisfies the system of equations (9) and (10). Substituting (10) in (9) yields

$$1 - \frac{x_i R_i}{\mathrm{e}^{R_i} - 1} \exp\left(-\frac{x_i}{\mathrm{e}^{R_i}} + \frac{\mathrm{e}^{R_i} - 1}{R_i \mathrm{e}^{R_i}} + n_i\right) = 0 \quad i = 1, 2 \qquad (23)$$

with $n_i = \frac{N_0}{P_j \sigma_{ji}^2}$. Thus, (10) and (23) provide an equivalent system to be satisfied by NE. In order to determine a NE we can proceed as in the case of the interference limited regime. Observe that, in this case, (23) depends on the system parameters and the other player strategy not only via x_i but also via n_i. Then, the general analysis feasible for any communication system in the interference limited regime is no longer possible and the existence and multiplicity of NEs should be studied independently for each communication system. In the following, we detail guidelines for this analysis.

From (23), it is possible to determine the best response in terms of rate of transmitter i to policy P_j of transmitter j. Conditions for the existence of such best response are detailed in the following statement.

Proposition 6 *Equation (23) admits positive roots if and only if*

$$1 - x_i \mathrm{e}^{-x_i + 1 + n_i} > 0. \qquad (24)$$

If (24) is satisfied, (23) admits a single positive root in the interval $(0, \log x_i)$, which corresponds to the best response in terms of rate to policy P_j of user j.

From the best responses in terms of rate, it is straightforward to determine the best response in terms of powers for the two players.

Acknowledgments

The authors thank Andrey Garnaev for useful suggestions and continuous support.

This work is partially supported by the French Research Agency (ANR) programme VERSO via the project Ecoscells and has been partially performed in the framework of the European research project SAPHYRE, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future.

References

[1] R. Berry and D. Tse, "Information theory meets game theory on the interference channel," in *Proc. of IEEE Information Theory Workshop (ITW)*, Jun. 2009, pp. 140–144.

[2] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicenced bands," *IEEE J. Select. Areas Commun.*, vol. 25, no. 3, pp. 517–528, Apr. 2007.

[3] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels," vol. 54, no. 7, pp. 2868–2878, Jul. 2008.

[4] G. Scutari and D. Palomar, "MIMO cognitive radio: A game theretical approach," vol. 58, no. 2, pp. 761–780, Feb. 2010.

[5] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," vol. 56, no. 10, pp. 5292–5295, Oct. 2008.

[6] S. V. Hanly and D. N. Tse, "Multi–access fading channels: Part II: Delay–limited capacities," *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2816–2831, Nov. 1998.

[7] S. Adlankha, R. Johari, and A. Goldsmith, "Competition in wireless systems via Bayesian interference games," in *http://arxiv.org/abs/0709.0516*, Sep. 2007.

[8] G. He, S. Gault, M. Debbah, and E. Altman, "Bayesian waterfilling game for fading multiple access channes," *submitted to Journal IEEE Transactions for Wireless Communications*, 2009.

[9] S. Akbarzadeh, L. Cottatellucci, and C. Bonnet, "Bayesian equilibria in slow fading OFDM systems with partial channel state information," Florence, Italy.

[10] L. Xiao, K. Avratchenkov, L. Cottatellucci, and A. Garnaev, "Equilibriums in slow fading interfering channels with partial knowledge of the channels," Eurecom, Tech. Rep. RR-11-246, Jan. 2011.

[11] J. Rosen, "Existence and uniqueness of equilibrium points for concave N-person games," vol. 33, no. 3, pp. 520–534, Jul. 1965.

[12] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge,MA: Cambridge University Press, 2005.