Direct Location Estimation using Single-Bounce NLOS Time-Varying Channel Models

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Abstract—Location estimation under Non-Line-of-Sight (NLOS) propagation conditions has been recognized as a very difficult task. In this contribution, a method that employs only one Base Station (BS) is proposed for tackling this problem. Its efficiency mainly stems from two factors. On one hand it exploits the information available in signal components traveling through different paths by considering proper channel modeling. On the other hand it combines the aforementioned spatial information with temporal information that is available in a dynamic channel. The source of the latter form of information is the Doppler shift. The performance of the method is further improved by considering the direct position and speed estimation from the received signal, rather than the common two-step approaches that are based on estimating channel-dependent parameters such as the Angle of Arrival (AOA) and/or the Time of Arrival (TOA), prior to localizing[1].

I. INTRODUCTION

Conventional geometrical techniques are based on the estimation, usually in more than one Base Stations (BSs), of location-dependent parameters, such as the Angle of Arrival (AOA), the Time of Arrival (TOA), the time difference of Arrival (TDOA), a combination of two of the above [1], [2] or the estimation of the Received Signal Strength (RSS) [3], [4]. After a subset of these parameters has been estimated, the location of the Mobile Terminal (MT) is determined by finding the candidate position that best fits the data.

The performance of the aforementioned two-step approach, has been proven to converge to the Cramer-Rao bound (CRB) for high Signal-to-Noise-Ratio (SNR) and sufficient number of data samples. However in wireless communications, high SNR is not always guaranteed. Furthermore, if the channel varies rapidly, the number of data samples that can be used in the estimation process is very limited. To localize efficiently at the low and moderate SNR regime or with short data records, the authors in [5], [6] proposed direct position determination (DPD). Direct position determination becomes feasible if a unique invertible mapping exists between the parameters of interest and the location-dependent parameters. Such a mapping is simple to derive when a Line-of-Sight (LOS) signal component exists in the received signal, thus, this assumption is always made in the existing literature.

In urban environments a LOS signal component rarely exists. The signal usually propagates in a rich scattering environment under strict NLOS conditions. The most common approach for localizing under these conditions is to try to mitigate the NLOS error. Two similar approaches, which are based on identifying the NLOS BSs, have been proposed for that purpose: Rejecting completely the measurements from NLOS BSs [7] or minimizing their effect by e.g. proper weighting [8]. Both of these approaches require the reception of the signal in many BSs, some of which must necessarily be linked through a LOS path with the MT. Therefore none of these is applicable to strict NLOS conditions. There exists, however, another total different approach to overcome the problem of NLOS reception. It is based on introducing an appropriate NLOS channel model [9], [10] and use its propagation characteristics to derive new equations that must be satisfied by the MT position's coordinates.

The method proposed herein falls into this category. It is based on the channel model employed in [10], which enables us to express the coordinates of the MT as a function of the location-dependent parameters, mentioned above. By doing so, we create the mapping required to implement the DPD. Our localization scheme does not require the reception of the MT's transmitted signal in more than one BS. It does, however, require multiple antennas at both ends to achieve high accuracy. To further improve the performance of our method, while slightly increasing its complexity, we propose to exploit the Doppler frequency shifts (due to the MT movement) and jointly estimate the MT speed along with its position. By considering an environment that changes dynamically, rather than a static one, we introduce one more dimension to the localization procedure, namely the (variation in) time. Thus, although two more unknown parameters (the speed components) need to be estimated, this new dimension offers a lot of information about the MT's position.

Notation: Throughout the paper, upper case and lower case boldface symbols will represent matrices and column vectors respectively. $(\cdot)^t$ will denote the transpose, $(\cdot)^*$ the conjugate and $(\cdot)^\dagger$ the conjugate transpose of any vector or matrix. For a $M \times N$ matrix $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_N]$, $vec(\mathbf{A}) = [\mathbf{a}_1^t, \ldots, \mathbf{a}_N^t]^t$ is a vector of length MN. For a square $M \times M$ matrix $\mathbf{A}$,

$diag(\mathbf{A})$ is a $M \times 1$ vector composed from its diagonal entries $a_{ii}, 1 \leq i \leq M$, while for a $M \times 1$ vector $\mathbf{a} = [a_1, \ldots, a_M]^t$, $diag(\mathbf{a})$ is an $M \times M$ diagonal matrix with $\mathbf{a}$'s entries along it's main diagonal. $[\mathbf{A}]_{(k:l,m:n)}$ is a submatrix $\mathbf{A}$ containing the common elements of rows $k, \ldots, l$ and columns $m, \ldots, n$. The symbols $\otimes$ and $\odot$ denote the Kronecker and Hadamard product. Finally for any random variable (r.v.) or vector $\mathbf{a}$, $\widehat{\mathbf{a}}$ will denote its estimated value and $\widetilde{\mathbf{a}} = \widehat{\mathbf{a}} - \mathbf{a}$ the estimation error.

II. CHANNEL MODEL

A. Geometrical Representation

The single bounce model (SBM) describes accurately NLOS propagation environments, despite the fact that it is very simple. Its wide applicability stems from the fact that in a physical propagation environment, the more bounces, the larger the attenuation will be, not only because the scatterer absorbs some of the signal's energy but also because more bounces usually implies a longer path length. Thus if a limited number of NLOS signal components with non-negligible energy arrive at the receiver, it is reasonable to assume that they have bounced only once. The existing SBM-based localization techniques consider a static propagation environment. We, on the other hand, are particularly interested in environments that change dynamically, due to the movement of the MT [2].

Let $\boldsymbol{\Phi}$, $\mathbf{T}$ and $\mathbf{F}_d$ denote the $N_s \times N_t$ matrices containing the AOA, the delays and the Doppler shifts respectively, and let $\boldsymbol{\psi}$ denote the $N_s \times 1$ vector containing the AOD. N_s is the number of the signal components arriving at the receiver and N_t is the number of the time instances (samples). Based on the single bounce model we can express the entries of the above matrices explicitly as a function of the MT coordinates, x_0 and y_0, its speed components (projection to the same axes), v_x and v_y and the set of the coordinates of the scatterers $\{x_{s_i}, y_{s_i}\}$. We can also express the entries of $\boldsymbol{\psi}$ as a function of $\{x_{s_i}, y_{s_i}\}$ and (x_{BS}, y_{BS}). With respect to figure 1 and using subscript li for the parameters at time instant t_l, $0 \leq l < N_t$ and corresponding to path (or scatterer) s_i, $1 \leq i \leq N_s$, the parameters of the SBM are given by:

$$\phi_{li} = \begin{cases} \tan^{-1} \frac{y_{s_i} - (y_0 + v_y dt_{l0})}{x_{s_i} - (x_0 + v_x dt_{l0})} &, \quad \frac{y_{s_i} - (y_0 + v_y dt_{l0})}{x_{s_i} - (x_0 + v_x dt_{l0})} > 0 \\ \pi + \tan^{-1} \frac{y_{s_i} - (y_0 + v_y dt_{l0})}{x_{s_i} - (x_0 + v_x dt_{l0})} &, \quad \frac{y_{s_i} - (y_0 + v_y dt_{l0})}{x_{s_i} - (x_0 + v_x dt_{l0})} < 0 \end{cases}$$
$$(1)$$

$$\psi_{li} = \psi_i = \begin{cases} \tan^{-1} \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} &, \quad \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} > 0 \\ \pi + \tan^{-1} \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} &, \quad \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} < 0 \end{cases}$$
$$(2)$$

$$\tau_{li} = \frac{1}{c}\sqrt{(y_{s_i} - (y_0 + v_y dt_{l0}))^2 + (x_{s_i} - (x_0 + v_x dt_{l0}))^2} + \sqrt{(y_{s_i} - y_{BS})^2 + (x_{s_i} - x_{BS})^2}$$
$$(3)$$

$$f_{d,li} = \frac{f_c}{c} \frac{v_x(x_{s_i} - (x_0 + v_x dt_{l0})) + v_y(y_{s_i} - (y_0 + v_y dt_{l0}))}{\sqrt{(y_{s_i} - (y_0 + v_y dt_{l0}))^2 + (x_{s_i} - (x_0 + v_x dt_{l0}))^2}}$$
$$(4)$$

[2]The extension to the more general case, where scatterers and/or the BS move too, is beyond the scope of this paper.
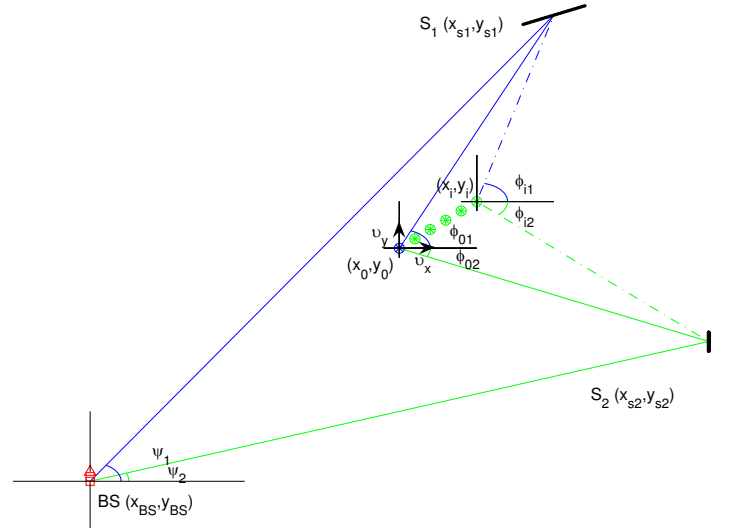

Fig. 1. Single Bounce model

where f_c is the carrier frequency, c is the speed of light and $dt_{l0} = t_l - t_0$ is the difference between two time instances. The above expressions are based on the assumption of a linear movement with constant speed, so that:

$$x_l = x_0 + v_x dt_{l0} , \; y_l = y_0 + v_y dt_{l0}$$
$$(5)$$

This assumption is valid if the total observation time $t_{tot} = (N_t - 1) \times \overline{dt}$ is small (e.g. fraction of a second), where $\overline{dt}$ is the average time between subsequent observations [3].

B. Input-Output Relationship

The input-output relationship of a $n_r \times n_t$ MIMO-OFDM system [4] for a time-variant (due to the MT movement), frequency selective channel is:

$$\mathbf{Y}(f_k, t_l) = \mathbf{H}(f_k, t_l)\mathbf{X}(f_k, t_l) + \mathbf{N}(f_k, t_l)$$
$$(6)$$

where $\mathbf{X}(f_k, t_l)$ is the $n_t \times N$ transmitted signal matrix, $\mathbf{Y}(f_k, t_l)$ is the $n_r \times N$ received signal matrix and $\mathbf{N}(f_k, t_l)$ is the $n_r \times N$ noise matrix, all at frequency $f_k, \forall 1 \leq k \leq N_f$ and time t_l. Throughout the rest of the analysis, the dependency on frequency and time will be denoted by the subscript kl for the sake of simplicity. For a NLOS environment that can be accurately described by the single bounce model, the channel matrix $\mathbf{H}_{kl}$ is given by [5]:

$$\mathbf{H}_{kl} = \frac{1}{\sqrt{N_s}} \sum_{i=0}^{N_s} \gamma_i e^{j2\pi f_{d,li} t_l} \mathbf{a}_R(\phi_{li}) \mathbf{a}_T^t(\psi_i) H_{TR,k} e^{-j2\pi f_k \tau_{li}}$$
$$= \mathbf{A}_{R,l}(\boldsymbol{\Gamma} \odot \mathbf{D}_{kl})\mathbf{A}_{T,l}^t$$
$$(7)$$

[3]By considering the average, we overcome the restriction of uniformly spaced measurement times.

[4]The choice of OFDM as transmission scheme stems from the fact that this technique transforms a frequency-selective wide-band channel into a group of frequency-flat parallel narrow-band channels.

[5]The proposed channel matrix representation is also valid for any NLOS environment where each AOA is linked with one AOD but not necessarily via a single scatterer.

where γ_i is the unknown complex amplitude of path i, $\mathbf{a}_R(\phi_{li})$ and $\mathbf{a}_T(\psi_i)$ are the $n_r \times 1$ and $n_t \times 1$ array responses of the receiver and the transmitter respectively for the signal component with AOA ϕ_{li} and AOD ψ_i and finally $H_{TR,k}e^{-j2\pi f_k \tau_{li}} = FT\{h_{TR}(\tau - \tau_{li})\}$ is the transfer function (Fourier Transform of the delayed impulse response) of the cascade of the filters at the transmitter's and receiver's front end. In the second equation we introduced the matrices:

$$\mathbf{A}_{R,l} \triangleq [\mathbf{a}_R(\phi_{l1}), \ldots, \mathbf{a}_R(\phi_{lN_s})] \tag{8}$$

$$\mathbf{A}_{T,l} \triangleq [\mathbf{a}_T(\psi_{l1}), \ldots, \mathbf{a}_T(\psi_{lN_s})] \tag{9}$$

$$\mathbf{\Gamma} \triangleq diag(\boldsymbol{\gamma}) \triangleq diag([\gamma_1, \ldots, \gamma_{N_s}]) \tag{10}$$

$$\mathbf{D}_{kl} \triangleq \frac{1}{\sqrt{N_s}} H_{TR,k} diag(\mathbf{d}_{kl}) \tag{11}$$

where

$$\mathbf{d}_{kl} \triangleq diag([e^{j2\pi(f_{d,l1}t_l - f_k\tau_{l1})}, \ldots, e^{j2\pi(f_{d,lN_s}t_l - f_k\tau_{lN_s})}]) \tag{12}$$

The model described above is a slightly modified -to make it more appropriate for estimation applications- version of the random matrix model introduced in [11]. It is a special case of the general information-theoretic model derived in [12] using the principle of Maximum Entropy. In that reference the authors proved that the complex path amplitudes should be i.i.d. Gaussian r.v. with 0 mean and variance 1 if a constant power term is introduced on the right hand side (r.h.s) of (7). Thus if the power term is integrated in the Gaussian r.v., their variance becomes σ_γ^2.

III. JOINT ESTIMATION OF SPEED AND INITIAL POSITION

We are interested in estimating jointly the MT's coordinates at time t_0, x_0 and y_0 and its speed components v_x and v_y directly from the received signal matrices $\mathbf{Y}_{kl}$. These two pairs of parameters (parameters of interest) compose a vector which we denote as $\mathbf{p}_{int} = [x_0, y_0, v_x, v_y]^t$. However since the AOA, the AOD, the delays and the doppler shifts depend also on the position of the scatterers, so will $\mathbf{Y}_{kl}$. Thus we need to estimate $\mathbf{p}_{int}$ in the presence of nuisance parameters which compose the vector $\mathbf{p}_{nuis} = [x_{s1}, y_{s1}, \ldots, x_{sN_s}, y_{sN_s}]^t$. This can be performed in two ways: Either jointly estimate $\mathbf{p}_{int}$ and $\mathbf{p}_{nuis}$ or derive the conditional p.d.f. of just $\mathbf{p}_{int}$ by integrating out $\mathbf{p}_{nuis}$. In this work we consider the first approach. Therefore our goal becomes to estimate the $(2N_s + 4) \times 1$ vector:

$$\mathbf{p} = [\mathbf{p}_{int}^t, \mathbf{p}_{nuis}^t]^t \tag{13}$$

Let $\boldsymbol{\phi} = vec(\boldsymbol{\Phi})$, $\boldsymbol{\tau} = vec(\mathbf{T})$ and $\mathbf{f}_d = vec(\mathbf{F}_d)$. Define the $N = (3N_t + 1)N_s$ vector containing all the channel-dependent parameters as:

$$\boldsymbol{\theta} = [\boldsymbol{\phi}^t, \boldsymbol{\psi}^t, \boldsymbol{\tau}^t, \mathbf{f}_d^t]^t \tag{14}$$

The entries of $\boldsymbol{\theta}$ depend on the entries of $\mathbf{p}$ through eq. (1)-(4). We can express the mapping of the vector $\mathbf{p}$ to the vector $\boldsymbol{\theta}$, along with its unique inverse, in a more compact way:

$$\boldsymbol{\theta} = g(\mathbf{p}) \ , \ \mathbf{p} = g^{-1}(\boldsymbol{\theta}) \tag{15}$$

This mapping allows us to do a direct position and speed estimation based on the received signal. Let $\mathbb{S}_{\mathbf{Y}} = \{\mathbf{Y}_{11} \ldots \mathbf{Y}_{N_f,N_t}\}$ and $\mathbb{S}_{\mathbf{H}} = \{\mathbf{H}_{11} \ldots \mathbf{H}_{N_f,N_t}\}$ be the set of all received signal matrices and the set of the corresponding channel matrices respectively. Define the following log-likelihood:

$$\mathcal{L} \triangleq \mathcal{L}(\mathbb{S}_{\mathbf{Y}}|\mathbf{p}) = \ln(f(\mathbb{S}_{\mathbf{Y}}|\mathbf{p})) \tag{16}$$

Using (15), $f(\mathbb{S}_{\mathbf{Y}}|\mathbf{p})$ can be derived as follows:

$$f(\mathbb{S}_{\mathbf{Y}}|\mathbf{p}) \equiv f(\mathbb{S}_{\mathbf{Y}}|g(\mathbf{p})) = f(\mathbb{S}_{\mathbf{Y}}|\boldsymbol{\theta})$$
$$= \int_{\mathbb{C}^{N_s}} f(\mathbb{S}_{\mathbf{Y}}|\boldsymbol{\theta}, \boldsymbol{\gamma}) f(\boldsymbol{\gamma}) d\boldsymbol{\gamma}$$
$$= \int_{\mathbb{C}^{N_s}} f(\mathbb{S}_{\mathbf{Y}}|\mathbb{S}_{\mathbf{H}}) f(\boldsymbol{\gamma}) d\boldsymbol{\gamma}$$
$$= \int_{\mathbb{C}^{N_s}} \prod_{k=1}^{N_f} \prod_{l=1}^{N_t} \left(\frac{1}{(\pi\sigma^2)^{n_r n_t}} e^{-\frac{1}{\sigma^2}|\mathbf{Y}_{kl} - \mathbf{H}_{kl}\mathbf{X}_{kl}|^2} \right)$$
$$\frac{1}{(\pi\sigma_\gamma^2)^{N_s}} e^{-\frac{1}{\sigma_\gamma^2}\boldsymbol{\gamma}^\dagger \boldsymbol{\gamma}} d\boldsymbol{\gamma} \tag{17}$$

The above integral was solved in [13] for $\sigma_\gamma^2 = 1$ and $\mathbf{X}_{k,l} = \mathbf{I}_{n_t}$, $\forall k, l$. The extension to the more general case is trivial and the result is given below:

$$f(\mathbb{S}_{\mathbf{Y}}|\mathbf{p}) \propto \det((\sigma_\gamma^2 \mathbf{V}\mathbf{V}^\dagger + \sigma^2\mathbf{I})^{-1})e^{-\mathbf{y}^\dagger(\sigma_\gamma^2 \mathbf{V}\mathbf{V}^\dagger + \sigma^2\mathbf{I})^{-1}\mathbf{y}} \tag{18}$$

where we have ignored the constant term and have introduced the $Nn_r N_f N_t \times N_s^2$ matrix $\mathbf{V}$ and the $Nn_r N_f N_t \times 1$ vector $\mathbf{y}$:

$$\mathbf{V} = [\mathbf{V}_{11}^\dagger, \ldots, \mathbf{V}_{N_f N_t}^\dagger]^\dagger \tag{19}$$

$$\mathbf{y} = [vec(\mathbf{Y}_{11})^\dagger, \ldots, vec(\mathbf{Y}_{N_f N_t})^\dagger]^\dagger \tag{20}$$

and each $Nn_r \times N_s^2$ submatrix $\mathbf{V}_{kl}$ is given by[6]:

$$\mathbf{V}_{kl} = (\mathbf{X}_{kl}^t \otimes \mathbf{I}_{\mathbf{n_r}})(\mathbf{A}_{T,l} \otimes \mathbf{A}_{R,l}) diag(vec(\mathbf{D}_{kl}))$$
$$= (\mathbf{X}_{kl}^t \otimes \mathbf{I}_{\mathbf{n_r}})(\mathbf{A}_{T,l} \otimes \mathbf{A}_{R,l})\mathbf{D}_{G,kl} \tag{21}$$

Define the conditional covariance matrix of the data vector $\mathbf{y}$ as:

$$\mathbf{C}_{\mathbf{y}|\mathbf{p}} \triangleq \sigma_\gamma^2 \mathbf{V}\mathbf{V}^\dagger + \sigma^2\mathbf{I} \tag{22}$$

Since $\mathbf{A}_{T,l}, \mathbf{A}_{R,l}$ and $\mathbf{D}_{G,kl}$ depend on $\mathbf{p}$, so does $\mathbf{V}$. Therefore $\mathbf{C}_{\mathbf{y}|\mathbf{p}}$ also depends on the parameters we need to estimate, although this dependency is not explicitly shown in (22). Substituting (22) in (18) and the result in (16) we get:

$$\mathcal{L} = -\ln(\det(\mathbf{C}_{\mathbf{y}|\mathbf{p}})) - \mathbf{y}^\dagger \mathbf{C}_{\mathbf{y}|\mathbf{p}}^{-1}\mathbf{y} \tag{23}$$

The Maximum Likelihood estimate of $\mathbf{p}$, denoted as $\widehat{\mathbf{p}}$, is given by maximizing the above log-likelihood:

$$\widehat{\mathbf{p}} = \underset{\mathbf{p}}{\operatorname{argmax}}\{\mathcal{L}\} \tag{24}$$

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[6]Due to the fact that $\mathbf{D}_{kl}$ is diagonal, we can perform a dimension reduction so that the size of $\mathbf{V}_{kl}$ becomes $Nn_r \times N_s$. However, the expression for $\mathbf{V}_{kl}$ becomes more complicated, so we will retain the general one.

IV. CRAMER-RAO BOUND

According to the Cramer-Rao Bound (CRB) for an unbiased estimator $\widehat{\mathbf{p}}$ of $\mathbf{p}$, the correlation matrix of the parameter estimation errors $\widetilde{\mathbf{p}}$ is bounded below by the inverse of the Fisher Information Matrix (FIM) $\mathbf{J}$ as shown below[7]:

$$R_{\widetilde{\mathbf{p}}\widetilde{\mathbf{p}}} = E\{(\widehat{\mathbf{p}} - \mathbf{p})(\widehat{\mathbf{p}} - \mathbf{p})^t\} \geq \mathbf{J}^{-1} \qquad (25)$$

The i', j' entry of the FIM is given by [14, eq.(8.35)]:

$$J_{i'j'} = -E\left\{\frac{\partial^2 \mathcal{L}}{\partial p_{i'} \partial p_{j'}}\right\}$$

$$= tr\left\{\mathbf{C}_{\mathbf{y}|\mathbf{p}}^{-1}\frac{\partial \mathbf{C}_{\mathbf{y}|\mathbf{p}}}{\partial p_{i'}}\mathbf{C}_{\mathbf{y}|\mathbf{p}}^{-1}\frac{\partial \mathbf{C}_{\mathbf{y}|\mathbf{p}}}{\partial p_{j'}}\right\} \qquad (26)$$

where

$$\frac{\partial \mathbf{C}_{\mathbf{y}|\mathbf{p}}}{\partial p_{i'}} = \frac{\partial(\sigma_\gamma^2 \mathbf{V}\mathbf{V}^\dagger + \sigma^2 \mathbf{I})}{\partial p_{i'}}$$

$$= \sigma_\gamma^2\left(\frac{\partial \mathbf{V}}{\partial p_{i'}}\mathbf{V}^\dagger + \frac{\partial \mathbf{V}^\dagger}{\partial p_{i'}}\mathbf{V}\right)$$

$$= \sigma_\gamma^2(\mathbf{V}'\mathbf{V}^\dagger + \mathbf{V}\mathbf{V}'^\dagger) \qquad (27)$$

and we have introduced $\mathbf{V}' = \frac{\partial \mathbf{V}}{\partial p_{i'}}$. This is a block matrix whose (k, l) submatrix $\mathbf{V}'_{(kl)}$ along with the partial derivatives required to construct it, are given at the bottom of this page. In those equations we have used the indicator function, defined as:

$$\mathbf{1}_{\mathbb{A}}(i') \triangleq \begin{cases} 1, & i' \in \mathbb{A} \\ 0, & i' \notin \mathbb{A} \end{cases} \qquad (32)$$

The partial derivatives of the entries of $\boldsymbol{\theta}$ with respect to the entries of $\mathbf{p}$ have been derived in our previous work and can be found in the appendix of [15], while for computing $\frac{\partial \mathbf{a}_T(\psi_i)}{\partial \psi_i}$ and $\frac{\partial \mathbf{a}_R(\phi_{li})}{\partial \phi_{li}}$, knowledge of the array geometry is required. Constructing $\mathbf{V}$ and $\mathbf{V}'$ from their submatrices, substituting their expression in (22) and (27) and then the result in (26) we get an expression for the FIM which is valid for any geometry of the arrays at the transmitter and the receiver.

V. NUMERICAL EXAMPLES

In this section we compute the best attainable performance (i.e. the CRB) of the proposed scheme in a picocell, since NLOS propagation most often occurs in such environments. The coordinates of the BS, the MT and the scatterers considered, are given in table I. The magnitude of the speed

[7]For matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \geq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is non-negative definite.
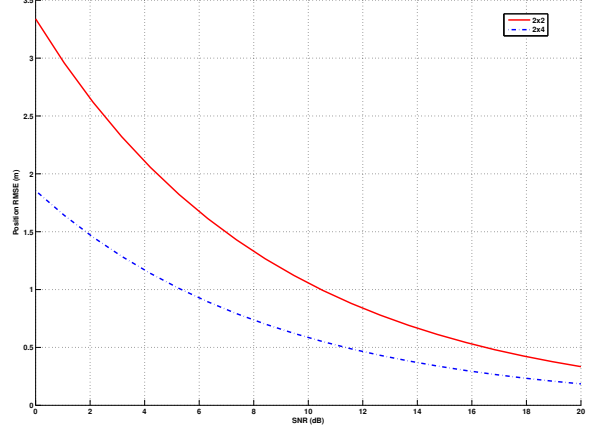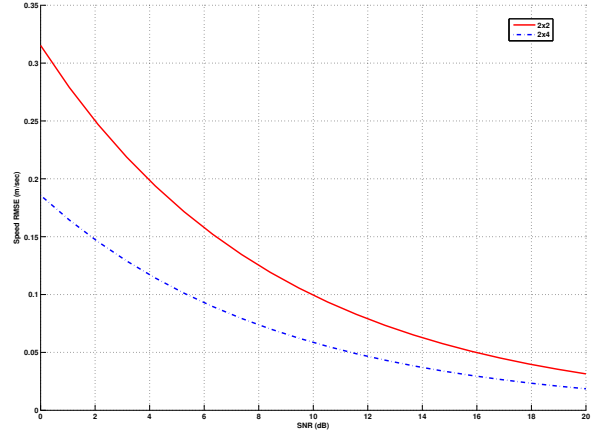

Fig. 2. Position Root Mean Square Error

Fig. 3. Speed Root Mean Square Error

of the MT is $|v| = 1.5$m/sec (average walking speed) and we average the results derived for 20 different directions of the speed, drawn indepedently from a uniform distribution with support region $[0, 2\pi]$. The $N_t = 40$ time samples are uniformly spaced and t_{tot} is slightly under 100msec. The transmitted signal is the (normalized-to-unit-energy) training matrix $\mathbf{X}_{kl} = \frac{1}{\sqrt{n_t}}\mathbf{I}_{n_t}, \quad \forall k, l$. The results hold for any transmitted signal that has been detected correctly before the

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$$\mathbf{V}'_{(kl)} = (\mathbf{X}_{kl}^t \otimes \mathbf{I}_{\mathbf{n_r}})\left[(\frac{\partial \mathbf{A}_{T,l}}{\partial p_{i'}} \otimes \mathbf{A}_{R,l} + \mathbf{A}_{T,l} \otimes \frac{\partial \mathbf{A}_{R,l}}{\partial p_{i'}})\mathbf{D}_{G,kl} + (\mathbf{A}_{T,l} \otimes \mathbf{A}_{R,l})\frac{\partial \mathbf{D}_{G,kl}}{\partial p_{i'}}\right] \qquad (28)$$

$$\frac{\partial \mathbf{A}_{T,l}}{\partial p_{i'}} = \mathbf{1}_{\{2i+3, 2i+4\}}(i')[\mathbf{0}_{n_t \times (i-1)}, \frac{\partial \mathbf{a}_T(\psi_i)}{\partial \psi_i}, \mathbf{0}_{n_t \times (N_s - i)}]\frac{\partial \psi_i}{\partial p_{i'}} \qquad (29)$$

$$\frac{\partial \mathbf{A}_{R,l}}{\partial p_{i'}} = \mathbf{1}_{\{1,\dots,4, 2i+3, 2i+4\}}(i')[\mathbf{0}_{n_r \times (i-1)}, \frac{\partial \mathbf{a}_R(\phi_{li})}{\partial \phi_{li}}, \mathbf{0}_{n_r \times (N_s - i)}]\frac{\partial \phi_{li}}{\partial p_{i'}} \qquad (30)$$

$$\frac{\partial \mathbf{D}_{G,kl}}{\partial p_{i'}} = \mathbf{1}_{\{1,\dots,4, 2i+3, 2i+4\}}(i')\frac{1}{\sqrt{N_s}}H_{TR,k}\left(t_l\frac{\partial f_{d,li}}{\partial p_{i'}} - f_k\frac{\partial \tau_{li}}{\partial p_{i'}}\right)diag([\mathbf{0}_{1\times(i-1)}, j2\pi e^{j2\pi(f_{d,li}t_l - f_k\tau_{li})}, \mathbf{0}_{1\times(N_s-i)}]) \qquad (31)$$

(x_{BS}, y_{BS})	(x_0, y_0)	(x_{s1}, y_{s1})	(x_{s2}, y_{s2})	(x_{s3}, y_{s3})
$(0,0)$m	$(30,20)$m	$(20,30)$m	$(5,5)$m	$(35,15)$m

estimation procedure[8]. We assume that both the transmitter and the receiver are equipped with Uniform Linear Arrays (ULA). The array response of the receiver's ULA to signal component i arriving at time l, is

$$\mathbf{a}_R(\phi_{li}) = [1, e^{j2\pi \frac{f_c}{c} d \sin(\phi_{li})}, \ldots, e^{j2\pi \frac{f_c}{c} d_r(n_r-1)\sin(\phi_{li})}]^t \tag{33}$$

and its partial derivative with respect to ϕ_{li} is

$$\frac{\partial \mathbf{a}_R}{\partial \phi_{li}} = j2\pi \frac{f_c}{c} d_r \cos(\phi_{li})[0, 1, \ldots, (n_r-1)]^t \odot \mathbf{a}_R(\phi_{li}) \tag{34}$$

where d_r is the distance between two adjacent antenna elements. Replacing d_r with d_t, ϕ with ψ and n_r with n_t, we get the array response (and the corresponding derivative) of the transmitter.

In figures 2 and 3 we plot the position and speed root mean square error (RMSE) respectively, versus the received SNR for a 2×2 and a 2×4 MIMO system. The SNR is defined as:

$$SNR = 10\log_{10}\left(\frac{E\{tr(\mathbf{HXX}^\dagger \mathbf{H}^\dagger)\}}{E\{tr(\mathbf{NN}^\dagger)\}}\right) = 10\log_{10}\left(\frac{\sigma_\gamma^2}{\sigma^2}\right) \tag{35}$$

where $\mathbf{H} = [\mathbf{H}_{11}, \ldots, \mathbf{H}_{N_f N_t}]$, $\mathbf{X} = [\mathbf{X}_{11}^t, \ldots, \mathbf{X}_{N_f N_t}^t]^t$ and $\mathbf{N} = [\mathbf{N}_{11}, \ldots, \mathbf{N}_{N_f N_t}]$. The position and speed RMSE are defined as:

$$RMSE_{\widetilde{x_0}, \widetilde{y_0}} = \sqrt{\sigma_{\widetilde{x_0}}^2 + \sigma_{\widetilde{y_0}}^2} = \sqrt{tr([\mathbf{J}^{-1}]_{(1:2,1:2)})} \tag{36}$$

$$RMSE_{\widetilde{v_x}, \widetilde{v_y}} = \sqrt{\sigma_{\widetilde{v_x}}^2 + \sigma_{\widetilde{v_y}}^2} = \sqrt{tr([\mathbf{J}^{-1}]_{(3:4,3:4)})} \tag{37}$$

In the figures we can notice that for the 2×4 system, $RMSE_{\widetilde{x_0}, \widetilde{y_0}}$ is less than $1m$ and $RMSE_{\widetilde{v_x}, \widetilde{v_y}}$ is less than $0.1m/sec$ for $SNR \geq 5.5dB$. The great enhancement in performance due to the increase in transmitting antennas (while keeping the same transmitting power) is obvious. Approximately the same enhancement can be alternatively achieved by doubling the time samples N_t, instead of n_t.

VI. CONCLUSIONS

We have proposed a localization scheme suitable for time-varying channels and NLOS reception. It is based on a geometrical representation and a corresponding statistical description of the channel. Its efficiency mainly stems from the fact that the variation in time due to the movement of the MT is a new source of information that can be exploited and integrated in classical geometrical localization techniques that

consider static channels. This comes at the cost of increased computational complexity, since the speed components need to be jointly estimated. Furthermore, due to the appropriate modeling, the scheme is able to implement the DPD approach and thus to ensure performance convergence to the CRB for small data records and/or low SNR. Instead of requiring the reception of the transmitted signal at multiple BS, the proposed method employs multiple antennas at the receiver and the transmitter to obtain more data and decrease the estimation error. Simulations reveal that it can achieve high accuracy under basic realistic assumptions.

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[8]Joint symbol detection and location estimation will be treated in future work.