DISTRIBUTED POWER ALLOCATION FOR COGNITIVE RADIO

Majed Haddad, Merouane Debbah and Aawatif Menouni Hayar

Mobile Communications Group, Institut Eurecom,
2229 Route des Cretes, B.P. 193, 06904 Sophia Antipolis, France
E-mail: {haddadm,debbah,menouni}@eurecom.fr

ABSTRACT

In this paper[1], we investigate the idea of using *cognitive radio* to reuse locally unused spectrum for communications. We consider a multiband/wideband system with two users in which the primary (licensed) user and the secondary (cognitive) user wish to communicate to the base station, subject to mutual interference. We introduce the notion of the virtual noise-threshold which represents a proxy for the primary user to allow cognitive communications. We determine, under the assumption that each user knows only his own channel, the acceptable interference level within a given quality of service. Moreover, we obtain a characterization of the distributed power allocation for each user, as well as the resulting virtual noise threshold. We prove that a cognitive user can vary its transmit power in order to maximize the sum capacity while maintaining a guarantee of service to the primary user.

1. INTRODUCTION

The recent boom in personal wireless technologies has led to an increasing demand in terms of spectrum resources. To combat this overcrowding, the Federal Communications Commission (FCC) has been investigating new ways to manage RF resources involving progressive redefinition of rules for accessing to the radio spectrum and posing several tasks in the management and in the sharing strategies for such a precious resource. Within this setting, the FCC has recently recommended [1] that significantly greater spectral efficiency could be realized by considering *cognitive radio* [2]. Such a scheme would define at least two classes of spectrum users. The first would be primary users who already possess a license to use a particular frequency. The second would be secondary (cognitive) users consisting of unlicensed users. Primary users would always have full access to the spectrum when they need it. Secondary users could use the spectrum when it would not interfere with the primary user. Cognitive radio systems offer the opportunity to improve spectrum utilization by detecting unoccupied spectrum bands and adapting the transmission to those bands while avoiding the interference to primary users. This novel approach to spectrum access is therefore based on reliable detection of primary users and adaptive transmission over a wide bandwidth. However, there are many challenges across all layers of a cognitive radio system design, from its application to its implementation [3].

In this work, we consider a TDD-uplink communication scenario in which the primary and the secondary user wish to communicate with the base station (BS), subject to mutual interference in a heterogeneous network where devices operate in a wideband/multiband context. One property of such systems is that, since the same frequency is used, the channel characteristics are nearly the same in both links, provided the channel does not change too rapidly. Under this scheme, we allow the secondary user to transmit simultaneously with the primary user as long as the primary user has not his quality of service (QoS) affected. The motivation of doing so in an environment where two senders share common resources is interference cancelation or mitigation. We derive the power allocations for each of the two users. We show that the overall system capacity can be considerably enhanced by considering cognitive communications.

The rest of the paper is organized as follows: In section 2, we present the channel model system. Section 3 describes the cognitive radio scenario. Section 4 details performance analysis of such systems as well as some simulation results. Section 5 concludes the paper.

2. THE CHANNEL MODEL

The baseband discrete-frequency model for uplink channel with two users as described in fig.(1) is:

$$y_{BS}^i = h_1^i \sqrt{P_1^i} S_1^i + h_2^i \sqrt{P_2^i} S_1^i + n^i, \tag{1}$$

where:

- h_k^i is the block fading process of user k for k = 1, 2 on the sub-band i for $i = 1, \ldots, N$,
- S_k^i is the symbol transmitted by user k on the sub-band i,
- P_k^i is the power control of user k on the sub-band i,

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- n^i is the additive gaussian noise at the ith sub-band.

We statistically model the channel h to be i.i.d distributed over the two fading gains and assume that $\mathbb{E}\left\{|h_k|^2\right\} = 1$. The additive gaussian noise n at the receiver is i.i.d circularly symmetric and $n \sim \mathcal{CN}(0, \sigma^2)$.

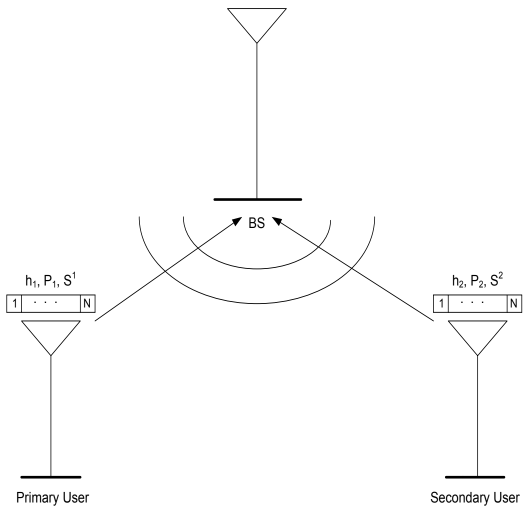

Fig. 1. Two-user cognitive radio uplink in a wide-band/multiband context.

3. THE COGNITIVE RADIO SCENARIO

In current cognitive radio protocol proposals, the device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum domain. Thus, a device transmits over a certain time or frequency band only when no other user does. In this work, the cognitive radio behavior is generalized to allow he secondary user to transmit simultaneously with the primary user as long as the level of interference with the primary user remains within an acceptable range, like in [6]. Specifically, we consider a communication scenario where devices operates in a wideband/multiband context where the two users can be jointly decoded using a successive interference cancelation (SIC) scheme [4].

On the other hand, in traditional systems (without cognition), when the primary user considers only the ambient noise level σ^2, he will maximize his rate by *selfishly* exploiting all resources through water-filling on this noise level [5]. According to this strategy, primary user leaves no resources for cognitive users to transmit. In our case, the primary user implicity over-estimates the noise level (which he considers as thermal noise plus interference) so as he leaves "space" for the secondary user. A key idea behind so doing is that, in any case, the primary user will not necessarily need all this rate. Our goal throughout this paper, is to show the feasibility of such approach as well as the capacity gain overall the system with respect to traditional communication systems. Moreover, one basic assumption throughout this contribution is that each user knows only his channel gains. Thus, when channel state information is made available at the primary user, he will adapt his transmission strategy relative to this knowledge by transmitting at the target rate less than the real data rate with an error-free transmission in order to maintain a guarantee of service. We obtain:

$$\log_2\left(1 + \frac{P_1^i \left|h_1^i\right|^2}{\sigma_v^2}\right) \leq \log_2\left(1 + \frac{P_1^i \left|h_1^i\right|^2}{P_2^i \mid h_2^i \mid^2 + \sigma^2}\right)$$

Reliable communication can therefore be achieved when the virtual noise threshold is higher than the cognitive interferer contributes, yielding:

$$\sigma_v^2 \geq P_2^i \mid h_2^i \mid^2 + \sigma^2, \qquad i = 1, ..., N \tag{2}$$

Accordingly, the virtual noise threshold has a double role:

(*i*) *it allows cognitive user to profit from primary user resources in an opportunistic manner,*

(ii) it maintains a guarantee of service to the primary user when cognitive communication is considered.

We will adopt this framework to *distributively* derive the optimum power allocation policies of each user and examine the variation of the sum capacity as function of the noise-threshold in order to obtain a characterization of the capacity gain by considering cognitive communication. In fact, although cognitive radios have spurred great interest and excitement in industry, many of the fundamental theoretical questions on the limits of such technology remain unanswered.

4. PERFORMANCE ANALYSIS

4.1. Short Term Analysis

For a given virtual noise-threshold σ_v^2, the maximum short term achievable rate that the primary user can obtain over the N sub-bands is given by:

$$C_{1,N} = \frac{1}{N}\sum_{i=1}^{N} \log_2\left(1 + \frac{P_1^i \mid h_1^i \mid^2}{\sigma_v^2}\right) \qquad (bits/s/Hz) \tag{3}$$

The optimal power allocation which maximizes the transmission rate at the primary user is solution of the optimization problem:

$$\max_{P_1^1,\ldots,P_1^N} C_{1,N},$$

subject to:

$$\begin{cases} \frac{1}{N}\sum_{i=1}^{N} P_1^i = 1, \\ P_1^i \geq 0, \end{cases} \tag{4}$$

In [5] authors looked at the problem of maximizing ergodic capacity subject to an average power constraint, and showed that the optimum power allocation follows from Shannon's principle of water-filling, namely [2]:

$$P_1^i = \left(\frac{1}{\gamma_0} - \frac{\sigma_v^2}{\left|h_1^i\right|^2}\right)^+, \qquad i = 1, ..., N \tag{5}$$

[2] $(x)^+ = \max(0, x)$.

Where γ_0 is the Lagrange's multiplier satisfying the short term average power in (4) with equality.
Let us now focus on the total capacity over the system when a cognitive user is allowed to profit from primary user resources. The expression of the short term sum capacity is given by:

$$C_{sum,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{P_1^i \mid h_1^i \mid^2 + P_2^i \mid h_2^i \mid^2}{\sigma^2}\right) \quad (6)$$

Secondary user offers the opportunity to improve the sum capacity over the system by reliably detecting primary user activity and adapting his transmission while avoiding the interference to the primary user by satisfying constraint in (2) with equality. The motivation of doing so is that if we allow the secondary user to transmit only if he is below a certain interference level, primary user should know the secondary user channel gains to decide at which interference level he should transmit in order to avoid outage. The secondary user power allocation is accordingly solution of the optimization problem:

$$\begin{cases} \frac{1}{N}\sum_{i=1}^{N} P_2^i = 1 \\ P_2^i \geq 0; \qquad i=1,\dots,N \\ \sigma_v^2 = P_2^i \mid h_2^i \mid^2 + \sigma^2; \qquad i=1,\dots,N \end{cases} \quad (7)$$

The secondary user power allocation that satisfies the constraints in (7) is therefore:

$$P_2^i = \frac{\sigma_v^2 - \sigma^2}{\left|h_2^i\right|^2}; \qquad i=1,\dots,N \quad (8)$$

Where the virtual noise threshold is given by:

$$\sigma_v^2 = \frac{N}{\sum_{i=1}^{N}\frac{1}{\mid h_2^i \mid^2}} + \sigma^2 \quad (9)$$

Thus, the secondary user inverses his channel fading in order to guarantee the QoS required to the primary user over the N sub-bands.

4.2. Asymptotic Performances

So far, we have derived power allocations for each user given a virtual noise threshold σ_v^2. Let us now investigate performances of such system when devices operates in a wide-band context, i.e. under the long-term constraint that $N \to \infty$. The goal here is to obtain a characterization of the resulting virtual noise threshold and, at the same time, prove the utility of using cognitive radio with respect to traditional communication systems.
By making N sufficiently large, each user can compute the virtual noise level *independently* based only the channel model statistic. In fact, within this setting, the long term average power constraint in (2) becomes:

$$\int_0^{\infty} P_2(t).p(t)dt = 1$$

Where $p(t)$ is the probability density function (p.d.f) of the channel model. By substituting P_2 by its expression in (7) we get the following interference constraint on σ_v^2:

$$\sigma_v^2 = \frac{1}{\int_0^{\infty}\frac{1}{t}.p(t)dt} + \sigma^2 \quad (10)$$

Notice here that the virtual noise threshold σ_v^2 depends only the SNR (through σ^2) and the channel statistics (through $p(t)$). Obviously, such an approach can be immediately translated into results for any non centered p.d.f in order to avoid the non-integrability in zero in (10).
Given this virtual noise level, we will study asymptotical performances of such a system in terms of the sum capacity when N is assumed to be infinite.

Theorem 1 *The sum capacity of cognitive systems using a virtual noise threshold as a proxy for the primary user performs always better than classical communication system (without cognition).*

Proof 1 *Let us firstly compute the expression of the sum capacity by making $N \to \infty$ when cognitive communications are possible. The expression in (6) becomes*

$$\begin{aligned} C_{sum,\infty} &= \int_0^{\infty}\log_2\left(1+\frac{P_1(t).t+P_2(t).t}{\sigma^2}\right).p(t)dt \\ &= \log_2\left(\frac{\sigma_v^2}{\sigma^2}\right)\int_0^{\gamma_0\sigma_v^2} p(t)dt + \int_{\gamma_0\sigma_v^2}^{\infty}\log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).p(t)dt \end{aligned}$$

Where γ_0 is the Lagrange's multiplier satisfying the long-term average power constraint, namely:

$$\frac{1}{\gamma_0}\int_{\gamma_0.\sigma_v^2}^{+\infty} p(t)dt - \sigma_v^2\int_{\gamma_0.\sigma_v^2}^{+\infty}\frac{p(t)}{t}dt = 1 \quad (11)$$

Similarly, we compute the sum capacity of a system where the primary user decides to maximize his rate selfishly. In other words, he will water-fill over the ambient noise level σ^2 and no resources will be left for cognitive users.

$$\begin{aligned} C'_{sum,\infty} &= \int_0^{\infty}\log_2\left(1+\frac{P_1(t).t}{\sigma^2}\right).p(t)dt \quad (12) \\ &= \int_{\gamma'_0\sigma^2}^{\infty}\log_2\left(\frac{t}{\gamma'_0.\sigma^2}\right).p(t)dt. \end{aligned}$$

Where γ'_0 is the Lagrange's multiplier satisfying the long-term average power constraint on σ^2. Numerical root finding is needed to determine different values of γ_0 and γ'_0. Our numerical results, show that γ_0, respectively γ'_0, increases as σ^2, respectively σ_v^2, decreases[3]. Now, let us compare the sum capacity in the two configurations. The difference between the two sum capacities computed above can be written as:

$$C_{sum,\infty} - C'_{sum,\infty} = \log_2\left(\frac{\sigma_v^2}{\sigma^2}\right)\int_0^{\gamma_0\sigma_v^2} p(t)dt + \Theta$$

[3] Since we have $\sigma_v^2 \geq \sigma^2$, we can conclude that $\gamma_0 \leq \gamma'_0$.

Where:

$$\Theta = \int_{\gamma_0 \sigma_v^2}^{\infty} \log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).p(t)dt - \int_{\gamma_0' \sigma^2}^{\infty} \log_2\left(\frac{t}{\gamma_0'.\sigma^2}\right).p(t)dt$$

Therefore, we have just to show that Θ *is positive. For the sake of simplicity, we make the assumption that we have* $\gamma_0.\sigma_v^2 \leq \gamma_0'.\sigma^2$. *We obtain*

$$\begin{aligned} \Theta \ = & \int_{\gamma_0 \sigma_v^2}^{\gamma_0' \sigma 2} \log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).p(t)dt + \\ & \int_{\gamma_0' \sigma^2}^{\infty} \underbrace{\left[\log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right) - \log_2\left(\frac{t}{\gamma_0'.\sigma^2}\right)\right]}_{\log_2\left(\frac{\gamma_0'.\sigma^2}{\gamma_0.\sigma_v^2}\right)}.p(t)dt \end{aligned}$$

Thus, under our assumptions, Θ *is always positive and the sum capacity of cognitive system performs always better than for traditional systems.*

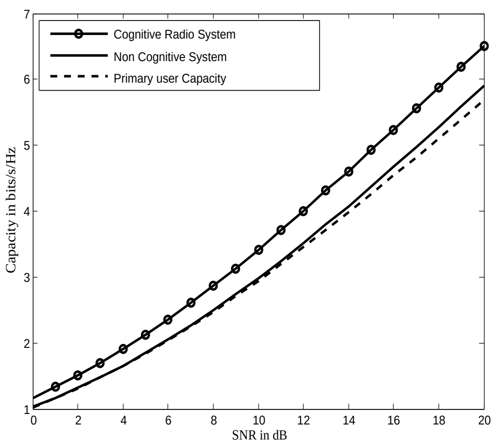

Fig. 2. Simulated capacities of the two configurations: with cognition and without cognition for N = 10.

Such a result shows the feasibility of allowing secondary users using locally unused spectrum for their transmissions with dynamic transmit powers and prove the fundamental constraint on the cognitive radios noise-threshold. In order to analyze the performance of our approach in terms of achievable rates, we model two rice channel models of mean = 1 to satisfy the constraint in (10). We plot capacities C_{sum} and C'_{sum} as function of the SNR. As mentioned before in theorem 1, it is clear that the cognitive system performs always better than for system where no cognition exists. Figure 2 also depicts primary user capacity expressed in (3). We remark that primary user capacity achieved in a cognitive system, where device water-fills over the virtual noise level σ_v^2, is always lower than for classical system, where device water-fills over the ambient noise level σ^2 especially at high SNRs of interest. Figure always better than for system where no cognition exists. Figure 3 illustrates the variation of the sum capacity for the two configurations. Notice here that by appropriately choosing the virtual noise threshold σ_v^2, one can increase the sum capacity over the system.

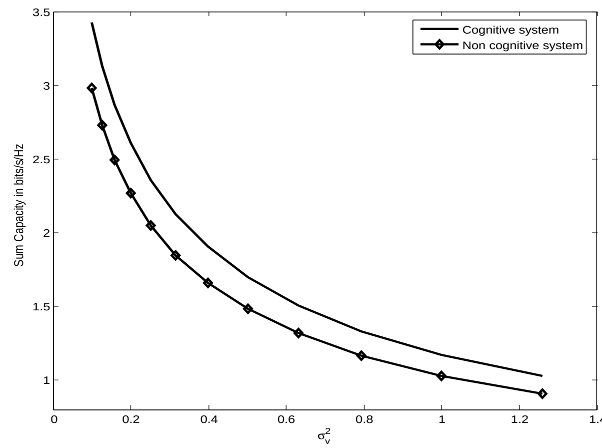

Fig. 3. Variation of the sum capacity as function of the virtual noise level for N = 100.

5. CONCLUSION

Using a virtual noise-threshold as a proxy for the primary user, we showed that a cognitive radio can vary its transmit power in order to maximize the sum capacity while maintaining a guarantee of service to the primary user. We characterized the variation of the sum capacity of such a system as function of the virtual noise-threshold σ_v^2. We also showed that one can improve the total capacity overall the system by allowing a single cognitive user to share primary user resources with respect to classical communication systems. As a future work, it is of major interest to generalize the problem to multi-user systems in order to characterize the sum capacity gain of such cognitive networks.

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