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Research Report RR-05-153

Optimal Constant-Window Backoff Scheme to Increase Throughput and Fairness of IEEE 802.11 DCF in Single-Hop Networks

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¹ Institut Eurécom's research is partially supported by its industrial members: Bouygues Télécom, France Télécom, Hitachi Europe, SFR, Sharp, ST Microelectronics, Swisscom, Thales, Cisco Systems, BMW Group Research & Technology - BMW Group Company .

Abstract

Existing backoff scheme's optimization of IEEE 802.11 DCF MAC protocol consider only saturated networks or asymptotic conditions. In real situations, traffic is bursty or streamed at low rates so that stations do not operate usually in saturated regime. In this work, we propose and analyze a backoff enhancement for IEEE802.11 DCF that requires information only about the network size and that is quasi-optimal under all traffic loads. We first analyze the performance of DCF multiple access scheme under general load conditions in single hop configuration and we provide an accurate delay statistics model that consider the self-loop probability in every backoff state. We prove then the short-term unfairness of the binary exponential backoff used in IEEE802.11 by defining channel capture probability as fairness metric. Motivated by the results on fairness, we introduce the constant-window backoff scheme and we compare its performance to IEEE802.11 DCF with Binary exponential backoff. The quasi-optimality of the proposed scheme is proved and numerical results show that it increases, both the throughput and fairness, of IEEE 802.11 DCF while remaining insensitive to traffic intensity. The analysis is then extended to consider the finite queueing capacity at nodes buffers using results from the delay analysis. NS2 simulations validate the obtained results.

1 Introduction

In the IEEE 802.11 standard for wireless LAN networks [1], the primary medium access scheme is called "distributed coordination function"(DCF) and it is based on a CSMA/CA protocol with binary exponential backoff (BEB) retransmission rules. Since the introduction of the standard, many works have been interested in the analytical evaluation of its performance; most of them were based on the model of Bianchi [2], and consider saturation throughput and delay analysis ([3–5] to cite few).

1.1 Queueing Analysis

In real networks, packets may be queued at nodes buffer before being handled by the MAC protocol, and typical data traffics are bursty or streamed at low rates so that stations do not operate usually in saturated regime. Recent works have addressed the finite load performance of IEEE802.11 DCF with queueing at nodes buffer (queues with infinite capacity) [6, 7] or with simplifying assumptions [8]. The analysis of queueing model of MAC protocols is a challenging task, and generally do not permit to obtain closed-form expressions of quantities of interest. In this work, we use a two-stage technique to analyze a queueing model of DCF protocol. In order to acquire closed-form expression of system performance, a Markov chain model is first used to analyze the non-queueing operation of the system. The traffic load in this case is modeled as a probability of having a packet to transmit q , this probability is taken into account whenever the protocol is able to handle a new packet. In this way, q allows us to consider the fact that packet arrivals may occur anytime during the operation of the system. From the non-queueing model, we obtain the service-time statistics corresponding to a given q . In the second phase, we consider a queueing model of the system with a given arrival process $\lambda(t)$ and queue length K . Thus, the probability of having a packet to transmit q corresponds to the probability of having at least one packet in the queue q_0 . In order to link the two models, we use a recursive algorithms that update the q value used in the Markov model to specify the service time statistics, to match the resulting q_0 from the queueing model.

1.2 Backoff Scheme Optimization

It is well recognized that the key optimization issue of random access protocols is the design of an optimal retransmission scheme that keeps access rate to the multiple-access channel around its capacity. Obviously, an optimal retransmission scheme must achieve this capacity under all network conditions and must be dis-

tributed. The optimality of the scheme depends on how accurate is the information that is has about the multiple access channel state.

IEEE802.11 DCF uses a BEB retransmission scheme. The BEB scheme has the advantage of being simple and does not require cooperation among users or any information about the channel state, it tries to blindly adapt the contention window to the channel congestion level based only on its experience, i.e., the contention window is increased in case of collision and it is reset to its initial value in case of success. Its performances however are shown to be sub-optimal, in term of the achieved throughput as it needs several attempts to find approximately the best contention window, and also in term of short-term fairness as it favors the first successful user to compete again for the channel with small contention window against potentially others users with much higher contention window. Works in [9,10] have derived specific fairness metric to illustrate this.

The enhancement of the DCF based BEB have been extensively addressed in the literature, the proposed schemes may be categorized into two classes:

1. Blind schemes: as in BEB, there is no need to sense the channel activity; the change of the contention window's length is made upon collision or success but in a different manner than BEB (MILD [11], FCR [12], EIED [13] to site few) in order to reach better the optimal backoff window and/or increase short-term fairness.
2. coherent schemes: here the optimization is made in order to dynamically adapt the contention window to meet directly some objective optimization condition. The objective condition is derived from an analytical model and its verification is made by measuring (estimating) some specific performance metrics, [14–17] to site few. Even if these schemes identify and try to reach an optimal operating point of the system, the way they update the backoff window is not optimal as in the blind schemes.

Early in the work of Bianchi [2], the notion of optimal backoff window that optimizes the saturation network throughput has been introduced. Unfortunately, the calculation of this optimal window requires information about the network size n and the average duration of collisions $E[T_{col}]$. Even if n could be easily obtained in single-hop network, channel activity sensing is required to estimate $E[T_{col}]$ in case of heterogeneous networks where users employ different physical rates and/or packet sizes.

As DCF provides equal long-term access rate to different users, several studies have shown that DCF is unable to fairly and efficiently manage heterogeneous networks [7, 14, 18–21]. As solution, time-based scheduling [19] have been shown to increase both the throughput and fairness of the MAC protocol.

In order to achieve trivially time-based scheduling with DCF, it is sufficient to normalize the packet duration by normalizing the packet-size/physical-rate ratio, i.e., each physical rate is to be used with a corresponding packet size in order to get unique packet duration on the channel and hence, a priori, fair input to the system. In this case, we can implement the optimal-window backoff scheme of [2] without estimating $E[T_{col}]$.

In this work, we consider backoff-window optimization issue of finite load single-hop networks based on the idea in [2]. In order to avoid estimating collision durations, we suppose that packet durations are normalized. Obviously, the optimal backoff-window in this case will depend also on the traffic load. However, we will show that is sufficient to use the saturation's optimal window under all loads to achieve nearly the maximum achievable throughput. This is an extended version of the paper in [22]. Our main contributions are:

- New analytical model to consider finite load performance of DCF without queueing at nodes buffer.
- Proof of the short term unfairness of the binary exponential scheme by using channel capture probability as fairness metric.
- Accurate delay statistics model considering self-loop probability on every backoff state.
- Introduction of the optimal constant-window backoff (OCB) scheme that maximizes the network throughput. The optimal window depends, among others, on the traffic load, and it is achieved only for arrival rates greater than a specific threshold. However, we prove in this work that the saturation optimal window is quasi-optimal under all traffic loads.
- Deep analysis of the operations of the BEB and OCB schemes with respect to load variations using numerical results. We show especially that OCB performs better than BEB, both in term of throughput and fairness, while remaining quasi-insensitive to traffic load.
- Analytical model to consider finite queueing capacity of nodes based on the delay statistics model of the non-queueing model. Using results on M/G/1/K queues, we will use a recursive algorithm to link the delay statistics produced by a given traffic load to a corresponding arrival process (Markovian in our case) and queue length.

The paper is organized as follows. In section II we introduce the analytical model, we derive the throughput and the delay statistics, and we show the unfairness of

the BEB retransmission scheme. In section III we introduce the optimal constant window backoff scheme and give bounds on performances loss when using only the saturation window for all arrival probabilities. The performances of the two schemes are then deeply analyzed in section IV. The finite capacity queueing model is given in section V, simulation results in section VI and concluding remarks are provided in section VII.

2 Binary Exponential Backoff Scheme

The analytical model we use is based on the work in [2] but extends it to consider general load performance (with backoff freezing and finite retry limit). We consider a network of n nodes evolving in single hop configuration. The key approximation of the Bianchi's model is to assume that the channel is busy with fixed probability p independently from the backoff counter value (equilibrium point analysis). Each node state is identified by its backoff window counter and backoff stage. The backoff counter and stage are modeled as a bidimensional discrete-time Markov process $(s(t), b(t))$ where $s(t)$ and $b(t)$ denote respectively the backoff stage and the backoff counter at time instant t . If the channel is busy the backoff counter is frozen for the duration of the current transmission. Otherwise, it is decreased when the channel is sensed again idle. Hence, transitions time of the Markov process depend on the current state of the channel. To alleviate this problem, a second approximation is made by defining an average time slot as the unit-time of the Markov chain. This unit-time is an average of the three possible time slot durations that correspond to successful transmission, collision or idle, weighted by their probability of occurrence:

$$T_{avg} = p_{idle}\sigma + p_{suc}T_{suc} + p_{col}T_{col} \quad (1)$$

σ is the idle slot duration. For the basic access mode, T_{suc} and T_{col} are given as

$$T_{suc} = 2\delta + H + E[P] + SIFS + Ack + DIFS \quad (2)$$

$$T_{col} = \delta + H + E[P] + EIFS \quad (3)$$

And for the RTS/CTS access mode

$$T_{suc} = 4\delta + H + E[P] + 3SIFS + RTS + CTS + ACK + DIFS \quad (4)$$

$$T_{col} = \delta + RTS + EIFS \quad (5)$$

p_{idle} , p_{suc} and p_{col} will be derived in the following.

When the backoff counter reaches 0 the node is allowed to transmit. In case of a collision, the node must double its contention window to reduce collision probability (binary exponential backoff). Otherwise it resets its contention window to its

initial value. The scheme defines also a maximum number $m + 1$ of retransmission trials after which the packet is dropped, and a maximum window's size order m' . Let $\pi_{i,j}$ denotes the steady state probability of node to be in backoff stage i with backoff counter at j . $i \in \{0..m\}$, $j \in \{0..W_i - 1\}$ and W_i denotes contention window value at stage i . According to the standard we have:

$$W_i = \begin{cases} 2^i W_0 & \text{for } i \leq m' \\ 2^{m'} W_0 & \text{for } i \geq m' \end{cases} \quad (6)$$

where W_0 is the initial value of the contention window.

To avoid channel capture, each node must wait a random backoff time after each successful packet transmission. We add then the new states $(-1, j)$, $j \in \{0..W_0 - 1\}$ to model node's state during inter-packets transmission (Inter-transmission back-off (ITB) states).

In order to consider the non-saturated regime we define q as the probability of having a packet to transmit (all nodes have the same q ¹), and to keep the analysis tractable we do not consider for the moment queueing at node's buffer (each node has at maximum one packet per time). In a queueing model, q corresponds to the probability of having at least one packet in the buffer.

Others works have addressed the performance analysis of 802.11 DCF under finite load conditions. In [6, 7], the authors analyzed the finite load performance of 802.11 considering queueing at nodes buffers. The analysis is more complex so they consider queues with infinite capacity. We mention also the work in [8] where the case of users with heterogeneous finite loads and with small buffers is analyzed. Using the assumptions of small buffers, the authors in [8] have modeled the arrival probability as the probability of having at least one arrival during the mean system time T_{avg} , which in fact remove the queueing effect as it is true only when the buffer size is equal to 1.

Here, we proceed differently, from the no-queueing model parameterized by the packet availability probability q , we derive the delay statistics and then we relate them to the finite capacity queueing model (section 5).

Fig. 1 illustrates the Markov chain model used for the no-queueing model. After a packet transmission (success or drop), a node may transit to the following states:

- $(0, 0)$: if it chooses 0 as backoff value and it has a packet to transmit

$$p\{(0, 0)|(i, 0)\} = \frac{(1-p)q}{W_0} \quad i \in \{0, m-1\} \quad (7)$$

$$p\{(0, 0)|(m, 0)\} = \frac{q}{W_0} \quad (8)$$

¹ extension to heterogeneous arrival case is straightforward [8]

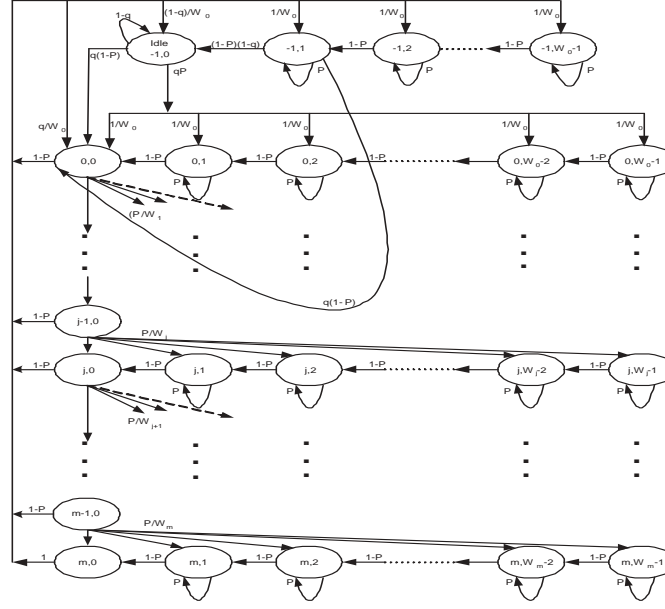


Fig. 1: Markov chain model

- $(-1, 0)$: if it chooses 0 as backoff value but has no packet to transmit.

$$p\{(-1, 0)|(i, 0)\} = \frac{(1-p)(1-q)}{W_0} \quad (9)$$

$$p\{(-1, 0)|(m, 0)\} = \frac{1-q}{W_0} \quad (10)$$

In this case, the node will stay in this state waiting for a new packet to transmit; *Idle* state.

- $(-1, j)$, $j \in \{1..W_0 - 1\}$: if it chooses j as backoff value.

$$p\{(-1, j)|(i, 0)\} = \frac{(1-p)}{W_0} \quad i \in \{0, m-1\} \quad (11)$$

$$p\{(-1, j)|(m, 0)\} = \frac{1}{W_0} \quad (12)$$

At the end of the ITB (state $(-1, 1)$), the node may transit to the $(0, 0)$ state if it has a packet to transmit. Otherwise, it goes to the *Idle* state.

$$p\{(0, 0)|(-1, 1)\} = (1-p)q \quad (13)$$

$$p\{(-1, 0)|(-1, 1)\} = (1-p)(1-q) \quad (14)$$

$$\pi_{0,0} \left[\sum_{i=0}^m p^i + \sum_{i=0}^m \sum_{j=1}^{W_i-1} \frac{(W_i-j)p^i}{W_i(1-p)} + \sum_{j=1}^{W_0-1} \frac{(W_0-j)p(1-q)}{W_0(1-p)} + \frac{1-q}{q} \right] = 1 \quad (22)$$

$$\pi_{0,0} = \begin{cases} \left[\frac{W_0(1-p)[1-(2p)^{m'+1}] + (1-2p)^2[1-p^{m'+1}] + 2^{m'} W_0(1-2p)[p^{m'+1} - p^{m+1}]}{2(1-p)^2(1-2p)} \right. \\ \quad \left. + (1-q) \left[\frac{p(W_0-1)}{2(1-p)} + \frac{1}{q} \right]^{-1} \right]^{-1} & m \geq m' \\ \left[\frac{W_0(1-p)[1-(2p)^{m+1}] + (1-2p)^2[1-p^{m+1}]}{2(1-p)^2(1-p)} + (1-q) \left[\frac{p(W_0-1)}{2(1-p)} + \frac{1}{q} \right]^{-1} \right]^{-1} & m \leq m' \\ \left[\frac{(W_0+1-2p)[1-p^{m+1}]}{2(1-p)^2} + (1-q) \left[\frac{p(W_0-1)}{2(1-p)} + \frac{1}{q} \right]^{-1} \right]^{-1} & m' = 0 \text{ (constant window)} \end{cases} \quad (23)$$

Transitions from the *idle* state occur at new packet arrival. If the medium is sensed idle during DIFS, the node proceeds directly with packet transmission and transits to the state $(0, 0)$. Otherwise, it executes the BEB scheme.

$$p\{(0, 0)|(-1, 0)\} = (1-p)q \quad (15)$$

$$p\{(0, j)|(-1, 0)\} = \frac{pq}{W_0} \quad j \in \{0..W_0-1\} \quad (16)$$

Solving the global balance equations leads to the following steady state probabilities

- for the last $m-1$ backoff stages:

$$\pi_{i,j} = \frac{(W_i-j)p^i}{W_i(1-p)} \pi_{0,0} \quad j \in \{1, W_i-1\} \quad (17)$$

$$\pi_{i,0} = p^i \pi_{0,0} \quad (18)$$

- for the inter-transmission backoff states:

$$\pi_{-1,j} = \frac{(W_0-j)}{W_0(1-p)} \pi_{0,0} \quad j \in \{1, W_0-1\} \quad (19)$$

$$\pi_{-1,0} = \frac{1-q}{q} \pi_{0,0} \quad (20)$$

- and for the first backoff stage

$$\pi_{0,j} = \frac{(W_0-j)p(1-q)}{W_0(1-p)} \pi_{0,0} \quad j \in \{1, W_0-1\} \quad (21)$$

The normalizing equation and the resulting steady state probability of being in state $(0, 0)$ are given in Eqs. (22,23). The probability of transmission in a given slot is then

$$\tau = \sum_{i=0}^m \pi_{i,0} = \frac{1-p^{m+1}}{1-p} \pi_{0,0} \quad (24)$$

Then the probabilities of busy, idle, success, and collision are given as

$$p = 1 - (1 - \tau)^{n-1} \quad (25)$$

$$p_{idle} = (1 - \tau)^n \quad (26)$$

$$p_{suc} = n\tau(1 - \tau)^{n-1} \quad (27)$$

$$p_{col} = 1 - p_{idle} - p_{suc} \quad (28)$$

and the throughput is defined as

$$Thrp = \frac{p_{suc}L}{T_{avg}} = \frac{p_{suc}L}{p_{idle}\sigma + p_{suc}T_{suc} + p_{col}T_{col}} \quad (29)$$

where L is the data packet length.

2.1 Delay Statistics

We define packet success delay as the time duration a packet lasts in the system since it is being handled by the MAC layer until the reception of acknowledgement of its successful reception.

A successful transmission may occur at one of the several backoff stages. The *average* time that a packet spends in the first backoff stage before its first transmission depends on whether the packet comes directly from the idle state or from ITB states. Conditioned on being in the first transmission stage $(0, 0)$, this time is

$$\begin{aligned} D_0 &= \left[1 - \frac{q(1-p)\pi_{-1,0}}{\pi_{0,0}} \right] \sum_{j=1}^{W_0-1} \frac{j}{W_0} D_B \\ &= [1 - (1-q)(1-p)] \frac{W_0-1}{2} D_B \end{aligned} \quad (30)$$

D_B denotes the average time that nodes spent in every backoff state. Many analysis of 802.11 delay take D_B equal to T_{avg} and ignore the self-loop probability p on every backoff state. In fact, D_B is geometrically distributed with parameter p and variates depending on the states of the $(n-1)$ remaining nodes

$$D_B = \sum_{k=0}^{\infty} p^k (1-p)(kT_B + \sigma) = \frac{pT_B + (1-p)\sigma}{1-p} \quad (31)$$

where T_B denotes the average slot duration seen by a node in backoff state when the channel is busy. Conditioned on channel busy probability p , T_B is

$$T_B = \frac{(n-1)\tau(1-\tau)^{n-2}[T_{suc} - T_{col}] + [1 - (1-\tau)^{n-1}]T_{col}}{p} \quad (32)$$

Similarly, for the other backoff stages, the *average* time that a packet spends in the stage i before its transmission

$$D_i = D_{i-1} + \frac{W_i-1}{2} D_B + T_{col} \quad i \in \{1 \dots m\} \quad (33)$$

D_{i-1} represents the time that the packet spends in the system until it's $(i-1)$ th. transmission, T_{col} the fact that the last transmission was not successful, and $\frac{W_i-1}{2}D_B$ is the average backoff time at the current backoff stage. Conditioned on starting transmission at the state $(0, 0)$, transmission success probability at the i th stage is

$$p_i^{suc} = \frac{\pi_{i,0}(1-p)}{\pi_{0,0}} = p^i(1-p) \quad i \in \{0 \dots m\} \quad (34)$$

The delay of a successful transmission can then be seen as a geometric random variable taking values in the set

$$\{D_i^{suc} = D_i + T_{suc}, i = 0 \dots m\}.$$

Alternatively, the *average* delay of packet drop is simply

$$E[D_{drop}] = D_m + T_{col} \quad (35)$$

With drop probability (conditioned on starting transmission at $(0, 0)$ state)

$$p_{drop} = p^{m+1} \quad (36)$$

Conditioned on effectively starting packet transmission (at state $(0, 0)$) success and drop probabilities sum to 1

$$\sum_{i=0}^m p_i^{suc} + p_{drop} = 1 \quad (37)$$

2.2 Short-Term Fairness

The use of exponential backoff retransmission scheme in 802.11 DCF leads to short-term unfairness. This is mainly because the scheme favors the first successful user to transmit again.

There exist several metrics to measure the fairness of a MAC protocol, the most popular is the one proposed by Jain et al. [23], but it can not be used for analytical purposes.

Many studies have then tried to characterize the short-term fairness issue by deriving specific fairness metrics [9], [10]. In [10], the authors define as metric the distribution of the number of inter-transmissions that other hosts may perform between two transmissions of a given host. They derive this metric for IEEE802.11 by considering the analytically tractable case of two nodes in saturation conditions and found surprisingly that the distribution of the number of inter-transmission K is independent of the contention window size. This means that changing the window size has no impact on fairness, so they conclude that unfairness of 802.11 DCF is not related to the use of the exponential backoff scheme.

In [10], the derivation of the distribution of K was possible by approximating the discrete uniform distribution by a continuous one. In doing so, the authors neglect the collision probability and so the analysis did not take into account the exponential backoff scheme which explains the misleading conclusions.

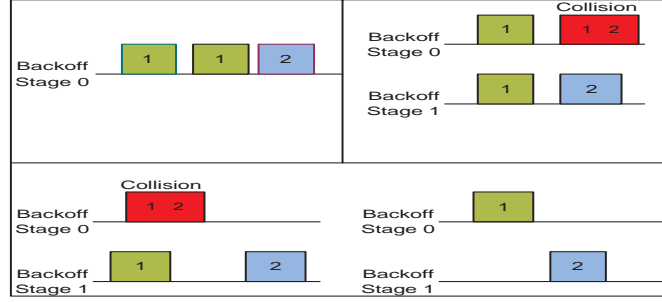


Fig. 2: Channel capture by node 1

To prevent analytical difficulties faced when deriving the distribution of K , we use as metric the channel capture probability, i.e., the probability that a node sends successfully and consecutively 2 packets. As this probability is smaller the scheme is fairer (for TDMA this probability is 0 as nodes use the channel alternatively). We derive this probability also only for the case of two nodes in saturation and we consider only two backoff stages. The goal is just to have an idea on the way the protocol performs in this simple scenario.

Consider two nodes 1 and 2, and let $w_{i,j}^k$ denote the k .th backoff window value chosen by node i when it enters backoff stage j . We denote the backoff window size at stage i by W_i and we suppose that the two nodes start simultaneously at stage 0.

The channel may be captured by node 1 only in the three following transmission cases: ‘11’, ‘1C1’ or ‘C11’ (Fig. 2). C denotes collision.

The event 11 represents a situation where node 1 chooses consecutively two backoff values $w_{1,0}^1$ and $w_{1,0}^2$ such that the backoff value $w_{2,0}^1$ chosen by node 2 is greater than $w_{1,0}^1 + w_{1,0}^2$. The probability of this event is

$$\begin{aligned}
 p(11) &= p(w_{1,0}^1 < w_{2,0}^1 \text{ \& } w_{1,0}^2 < w_{2,0}^1 - w_{1,0}^1) \\
 &= \sum_{i=0}^{W_0-1} \sum_{j=i+1}^{W_0-1} \sum_{k=0}^{j-i-1} \frac{1}{W_0^3} = \frac{W_0^2 - 1}{6W_0^2}
 \end{aligned} \tag{38}$$

We can see that this probability increases with increasing W_0 and it’s independent of the choice of W_1 .

The event 1C1 represents a situation where the backoff values chosen by node 1 at the first backoff stage, and then after a collision at the second backoff stage, are smaller than those of node 2. The probability of this event is

$$\begin{aligned}
p(1C1) &= p(w_{10}^1 < w_{20}^1 \text{ \& } w_{20}^1 = w_{10}^1 + w_{10}^2 \text{ \& } w_{11}^1 < w_{21}^1) \\
&= \left[\frac{1}{W_0} \sum_{i=0}^{W_0-1} \sum_{j=i+1}^{W_0-1} \frac{1}{W_0^2} \right] \left[\sum_{i=0}^{W_1-1} \sum_{j=i+1}^{W_1-1} \frac{1}{W_1^2} \right] \\
&= \frac{(W_0-1)(W_1-1)}{4W_0^2W_1} \tag{39}
\end{aligned}$$

We observe that this probability decreases with increasing W_0 (collision probability is decreased) and increases with increasing W_1 .

The third event represents a situation where after a collision, node 1 succeeds to transmit first its packet, then it goes-back to the first backoff stage and transmit again before node 2. The probability of the third event is

$$\begin{aligned}
p(C11) &= p(w_{10}^1 = w_{20}^1 \text{ \& } w_{11}^1 < w_{21}^1 \text{ \& } w_{10}^2 < w_{21}^1 - w_{11}^1) \\
&= \left[\frac{1}{W_0} \sum_{i=0}^{W_1-1} \sum_{j=i+1}^{W_1-1} \sum_{k=0}^{j-i-1} \frac{1}{W_0W_1^2} \right] \\
&= \begin{cases} \frac{W_1^2-1}{6W_0^2W_1} & W_1 \leq W_0 \\ \frac{3W_1^2+W_0^2-3W_0W_1-1}{6W_0W_1^2} & W_1 > W_0 \end{cases} \tag{40}
\end{aligned}$$

We observe again that this probability increases with increasing W_1 . In fact, after a collision the first successful node has a smaller contention window than the other node so it has more chance to retransmit again.

The channel capture probability is the sum of probabilities of the last three events. As we have seen, the channel capture probability increases with increasing W_1 (Fig. 3) which means that binary exponential backoff scheme is less fair than constant backoff scheme ($W_1 = W_0$). We observe also that BEB is fairer for increasing size of the initial backoff window. The result is for the case of two nodes but give a general idea on the behavior of the protocol. Intuitively, if the network size increases, the collision probability increases, and so, the probability that nodes will alternate transmissions after collision decreases as they have different windows. The same argument can be used to prove the same behavior for increasing number of backoff stages. We are then facing a capacity-fairness tradeoff; after a collision, if the contention window is increased, the system becomes unfair, but in the same time the collision probability is decreased.

Historically, the BEB scheme was introduced to blindly adapt the contention window to the traffic load in order to reduce collisions. Recently, it was shown in [24] that the BEB achieves a success probability of $\ln 2/2$ which is lower than

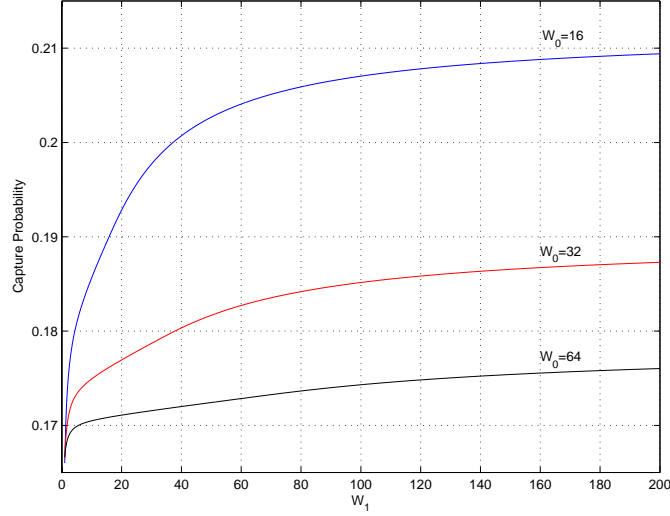


Fig. 3: Channel capture probability Vs. W_1

the capacity of a constant backoff scheme (e^{-1} for slotted Aloha with uniform retransmission). It is then legitimate to think about a constant backoff scheme that blindly adapt or that is insensitive to traffic load.

3 Optimal Constant-Window Backoff Scheme

Motivated by the results on short-term unfairness of BEB, we analyze in depth the case of constant backoff window. In this case, the backoff window must be optimized to maximize the throughput and must be fixed to not decrease fairness. The optimal backoff window can be seen as the transmission probability τ_{op} , below which the channel utilization is reduced due to high probability of idle slots and above which reduction of throughput is due to high collision probability. The goal of the optimization is then to adapt the backoff window to achieve this τ_{op} . Obviously, under general load conditions, the backoff window must be optimized with respect to traffic intensity (q). However, it is also obvious that the $\{\tau_{op}\}$ will not be achieved for small arrival rate ($q \leq q_t$, q_t is a threshold on arrival rate) even with the minimal backoff window ($W_0 = 1$). For this reason, we propose in this work to use the optimal backoff window of the saturated regime W_{op}^s for all arrival rates. The intuition behind this choice is that below q_t the system is lightly loaded so that the probability of going into backoff is very small and thus the effect of us-

ing a large W is minimal. Above q_t , the loss incurred by using a backoff window $W_0 = W_{op}^s \geq W_{op}$ is due to the fact that idle slot probability is higher than the optimal one, but in this case, the packet collision probability is lower than the optimal one, since in CSMA system the idle slot duration is small compared to the collision duration, the loss in the achieved throughput is small. In the following, we derive first the optimal transmission probability τ_{op} and the arrival rate threshold q_t . Once q_t identified, we show that for arrival rates below q_t almost all transmissions succeed without involving the backoff scheme, and for $q \geq q_t$ we give an upper bound on the throughput loss.

3.1 Derivation of τ_{op} and W_{op}^s

When we differentiate the $Thrp$ with respect to τ , we find that is maximal for transmission probability τ_{op} verifying²

$$\tau_{op} = \frac{\alpha - (1 - \tau_{op})^n}{\alpha n} \quad \text{where} \quad \alpha = \frac{T_{col}}{T_{col} - \sigma} \quad (41)$$

From Eq.(23) we have for the constant backoff case ($n' = 0$) in saturation conditions ($q = 1$)

$$\tau = \frac{2(1-p)}{(W_0 + 1 - 2p)} = \frac{2(1-\tau)^{n-1}}{[W_0 - 1 + 2(1-\tau)^{n-1}]} \quad (42)$$

The saturation optimal fixed backoff window is then

$$W_{op}^s = 1 + \frac{2(1-\tau_{op})^n}{\tau_{op}} \quad (43)$$

3.2 Derivation of q_t

We look now under which condition on q the τ_{op} could not be achieved even with the minimal allowed value of the backoff window $W_0 = 1$ (no backoff³). From Eq. 23 we have for $W_0 = 1$

$$\tau = \frac{q(1-p)}{qp + 1 - p} \quad (44)$$

After some algebra we find that the situation of $\tau \leq \tau_{op}$ is possible for

² The existence and uniqueness of τ_{op} can be simply verified [2]

³ We take $W_0 = 1$ only for analytical purpose, in real system the lowest value of W_0 we may take is 2

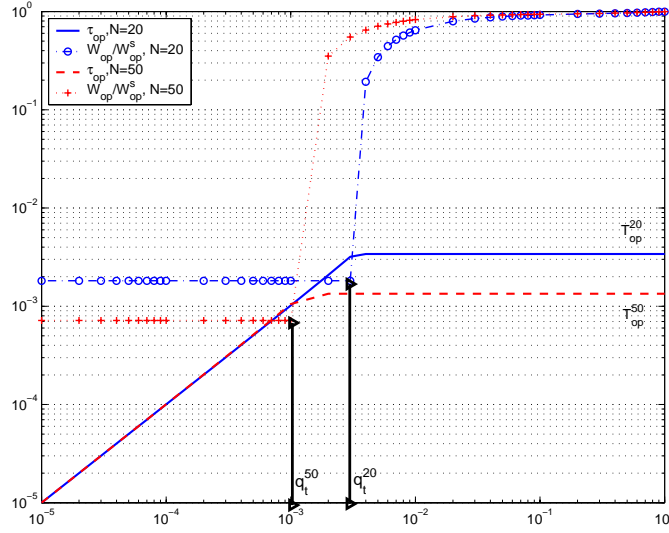


Fig. 4: Optimal transmission probabilities and the corresponding optimal backoff windows (normalized to the saturation optimal window) vs. arrival rates

$$q \leq q_t = \frac{\tau_{op}(1 - p_{op})}{1 - p_{op} - \tau_{op}p_{op}} \quad (45)$$

$$\text{Where } p_{op} = 1 - (1 - \tau_{op})^{n-1} \quad (46)$$

In Fig 4, we plot the Optimal transmission probabilities and the corresponding optimal backoff windows Vs arrival rates. We can see that for arrival probabilities $q \leq q_t$ the achieved transmission rates are below the optimal ones even with back-off window equal to 1. We then say that the system is in lightly loaded regime. Above q_t , τ_{op} is achieved by increasing the backoff window. We observe also that the optimal backoff window increases, in a first phase, exponentially and then, in a second phase, slowly converges to the saturation optimal window. During the first phase of increase we say that the system is in transition regime while during the second phase it is in saturation regime.

In the following, we give bounds on throughput loss when using the saturation optimal window under all load rather than the exact optimal window that take into account the value of traffic load.

3.3 Loss in System Performances

3.3.1 Case of $q \leq q_t$

As we have said before, in this case the system is lightly loaded and almost all transmissions are successful without backoff. To see this we can express the transmission success probability outside the backoff state as

$$P_{suc}^{NB} = n\pi_{-1,0}q(1-p)^2 = n(1-q)(1-p)^2\pi_{0,0} \quad (47)$$

While total transmission success probability is given as

$$p_{suc} = n\tau(1-p) = n(1-p^{m+1})\pi_{0,0} \quad (48)$$

As $q \leq q_t$, then we have $\tau \leq \tau_t$, where τ_t is the transmission probability corresponding to traffic load q_t . Thus, we can lower bound the ratio of P_{suc}^{NB} over p_{suc} as follow

$$\frac{P_{suc}^{NB}}{p_{suc}} \geq (1-q_t)(1-p_t)^2 \quad (49)$$

$$\text{Where } p_t = 1 - (1 - \tau_t)^{n-1} \quad (50)$$

In Fig. 5 we plot this lower bound Vs. network size and we can see that about 94% of transmissions success occurs without backoff. We conclude then that the use of the saturation optimal window in this case has almost no effect on system performances.

3.3.2 Case of $q \geq q_t$

In this case the τ_{op} is achieved if the backoff window is optimally adapted to the arrival rate. The maximum system throughput is then achieved. Using Eq. 41 we can express this maximum throughput as follow

$$Thrp_{max} = \frac{(1-p_{op})E(P)}{(1-p_{op})T_{suc} + p_{op}T_{col}} \quad (51)$$

From this last expression of the maximal throughput we can deduce that the optimal operation of the protocol is similar to having only one node in saturation condition who succeed its transmissions with probability $1 - p_{op}$ and fails with probability p_{op} .

Now, we want to measure the loss in the achieved throughput if we do not use the optimal window to achieve τ_{op} but only the saturation optimal window. As

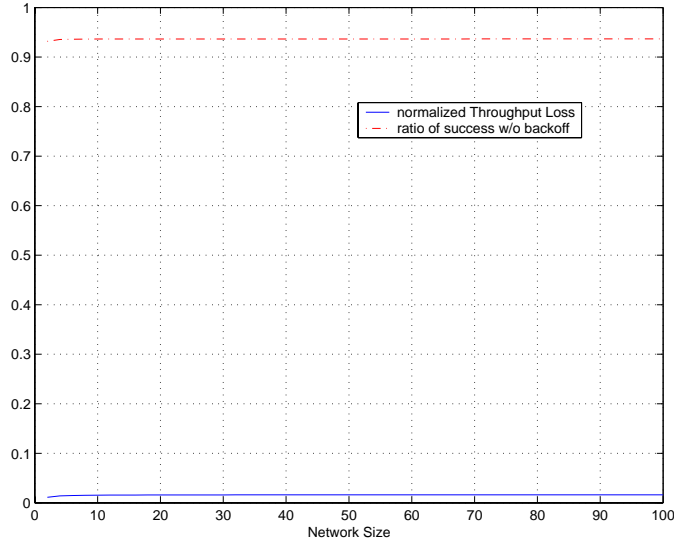


Fig. 5: Bounds on system performances

δ	σ	<i>SIFS</i>	<i>DIFS</i>	<i>EIFS</i>	<i>H</i>	$E[P]$	<i>RTS/CTS</i>	<i>ACK</i>
$1\mu s$	$20\mu s$	$10\mu s$	$50\mu s$	$364\mu s$	416	8184	352	304

Tab. 1: Parameter set used for numerical results

$q \geq q_t$, we have $\tau \geq \tau_q$, we can thus upper bound the normalized throughput loss as follow

$$\frac{Thrp_{max} - Thrp}{Thrp_{max}} \leq \frac{Thrp_{max} - Thrp_t}{Thrp_{max}} \quad (52)$$

In Fig. 5 we plot this bound Vs. networks size and we find that the loss does not exceed 1.6%.

4 Numerical Results

In this section, we compare the performance of the IEEE802.11 DCF based BEB with the proposed optimal constant backoff (OCB) scheme. Table 1 summarizes the parameters used for our numerical results.

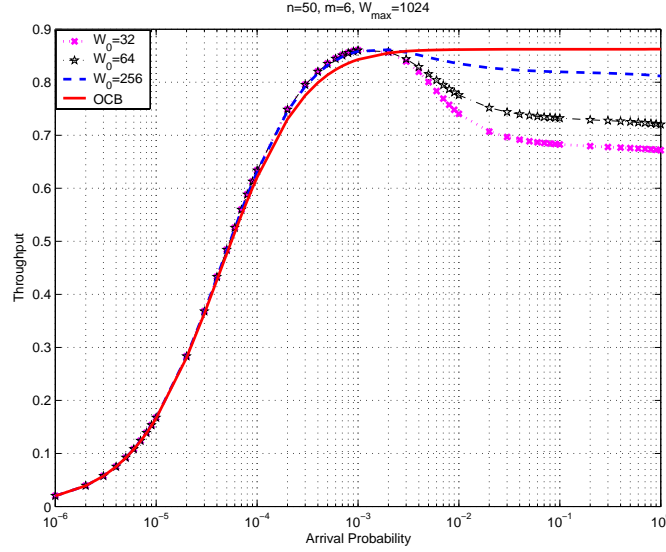


Fig. 6: Throughput

4.1 Throughput

Fig. 6 shows the achieved throughput Vs. packet arrival probability for network of size $n = 50$. The optimal window for OCB scheme in this case is 1392 slots. We consider multiple BEB cases with different initial backoff window $W_0 = 16, 64, 256$. We see then that during the lightly loaded regime ($q \leq 10^{-3.5}$ in this case), both OCB and BEB (independently from W_0) perform similarly and increase their channel utilization with increasing q . During the transition regime ($10^{-3.5} \leq q \leq 10^{-2.8}$), we observe that the BEB throughput is slightly higher than the OCB one. Finally in the saturation regime ($q \geq 10^{-3.5}$), and depending on W_0 , the throughput achieved by BEB scheme decreases and then saturates, while the OCB throughput saturates at a higher value.

To understand the operation of the two schemes, we plot in Fig. (7,8) the repartition of success probability (is the successful transmissions occur from *idle* state or from backoff states?), the collision probability and the idle probability Vs. packet arrival probability q . We consider the case of $W_0 = 64$ for BEB. For both schemes we observe that during the 1st phase, success and collision probabilities increase with increasing load while idle probability remains almost equal to 1 which means that the system is lightly loaded. As result, all success is almost from the *idle* state which means that almost all packets are transmitted directly at their arrivals without any backoff delay. In the 2nd phase, for the BEB scheme, the collision probability

continues to increase with load while idle probability starts to decrease seriously. BEB begins then to have significant transmission success from the backoff state while success from the *idle* state saturates. At the end of this phase, the two success probabilities are equal. The same phenomena is observed for OCB, except the idle probability that decreases also but remains close to 1, and a less significant success from backoff states which means that almost all success is still produced at *idle* state.

To explain this and how the difference in success probability repartition produces the small difference in the channel utilization, we can say that during this transition phase, the probability of busy slot at packet arrival increases for the two schemes. They start then to execute occasionally their backoff procedures. As the BEB scheme begins with a relatively small value of W_0 , its busy slot probability is bigger than for OCB (the users are not delayed for a long time), so it enters more frequently into backoff states, but as the system is still lightly loaded, it succeeds its transmission without excessive backoff delay (the panel of backoff windows (from W_0 to W_{max}) is sufficient to statistically multiplex efficiently all access demands). The OCB scheme operates differently; as its backoff window is bigger (1392), its busy slot probability is smaller than for BEB (high idle probability), so it enters less frequently the backoff state. But in the same time, as the system is lightly loaded, even if the system delays far enough the unlucky users who find the system busy at their packet arrival, the channel is not used frequently during this time which explains the small loss in channel utilization.

During the 3rd phase, the total success probability of the two schemes saturate as well as idle and collision probabilities (and so the throughput). For the BEB, success from backoff states continues to increase with load becoming the only significant source of transmission success. While for the OCB scheme, success from backoff states becomes significant only at values of q approaching 1. The degradation of throughput of BEB can be seen as a failure of the scheme to adapt its window to access demands (high collision probability). The OCB scheme is more efficient during this phase, as its backoff window is tailored for a saturated regime. Even if it continues to delay unlucky user for a longer time than BEB, the channel utilization get higher as the load increases.

Another important observation is that even if BEB achieves higher success probability than OCB, the resultant throughput is lower! This gives us a more precise idea on the philosophy of the scheme; In fact, OCB fixes the optimal window in order to keep transmission probability in an optimal level. At this optimal level loss due to idle slots is equal to loss due to collision. Below this optimal level, idle slot probability increases while success and collision probabilities decrease. Above the optimal transmission level, success increases but also collisions. In carrier sense multiple access scheme, idle slot duration is shorter than collision, the scheme tries

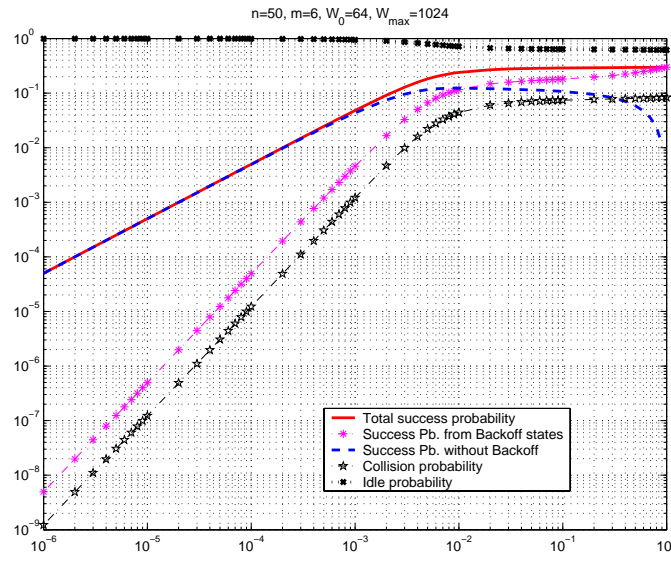


Fig. 7: Repartition of Success probability of the BEB Scheme

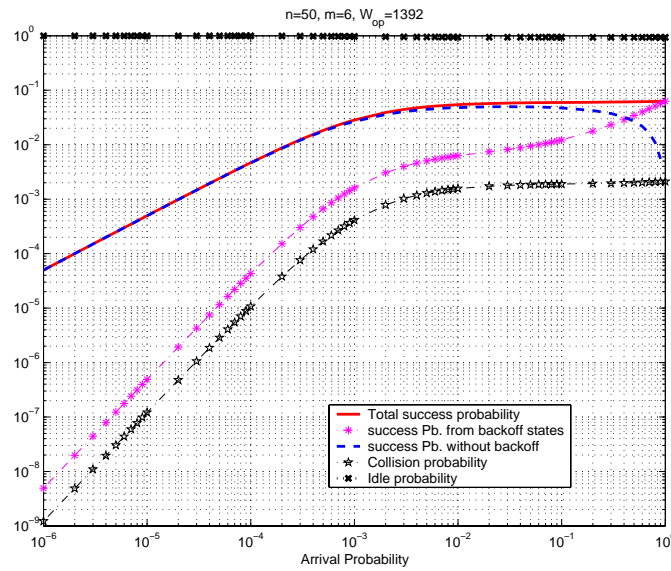


Fig. 8: Repartition of Success probability of the OCB Scheme

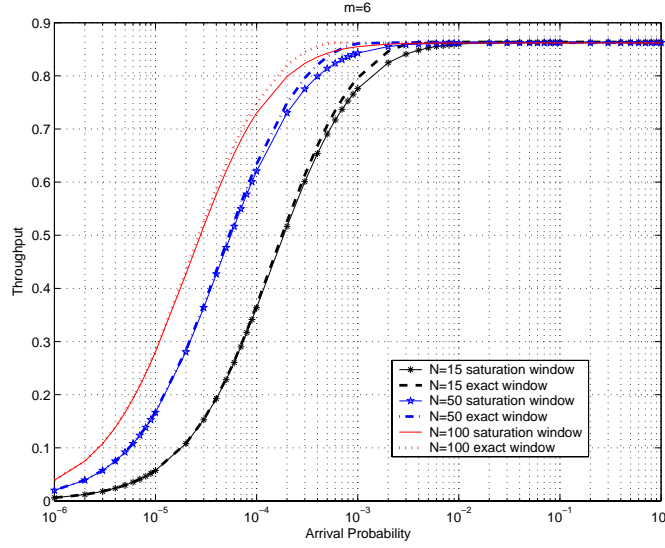


Fig. 9: Throughput for Optimal Exact Window and Optimal Saturation Window

then to equalize the duration of idle and collision events which explains the large value for the contention window and so the smaller success probability.

OCB seems then to operate at optimal level regardless of traffic intensity except during the transition phase. In Fig. 9, we compare the throughput achieved by OCB to the one achieved by exactly optimizing the backoff window to the traffic load q . As predicted by the bounds in section (3.3), we observe that the loss of OCB is small for all network size considered, and is located on a small interval that corresponds to the transition phase.

To illustrate better the superiority of OCB over BEB, we plot in fig. 10 the achieved throughput of the two schemes in saturation vs. network size. We observe that OCB performs better than BEB at all network size. We observe also that BEB operates differently depending on its initial backoff window value. We can see that every value of W_0 has only a limited interval of network sizes where it performs optimally which shows the inability of BEB to adapt efficiently the backoff window to the access demands.

4.2 Delay & Fairness

Fig. 11 depicts the normalized achieved delay (to packet transmission time T_{suc}) Vs. packet arrival probability. We observe a logical behavior with respect to the throughput, i.e., no excess delay in the non-backoff regime, delay of OCB

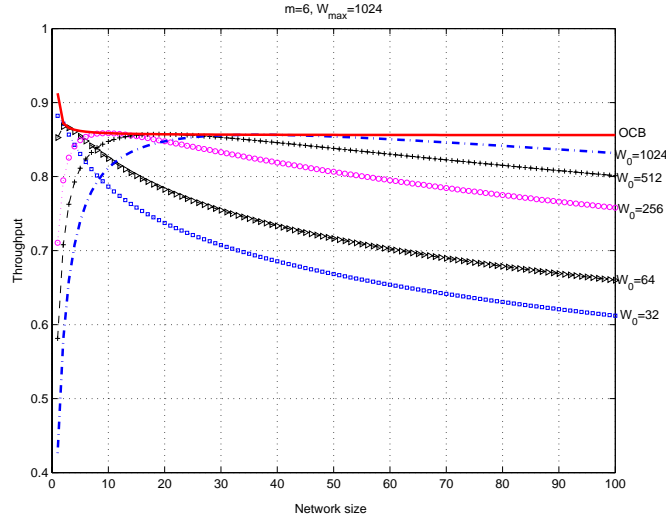


Fig. 10: Throughput Vs. Network size

slightly greater than BEB in the transition regime and lower in saturation regime. Moreover, we can see that OCB packet's mean delay at saturation approximates $50 * T_{suc}$, which is the delay of a pure TDMA scheme with 50 users in saturation.

To illustrate the BEB unfairness, we use the Jain's fairness index relative to the delay. The Jain's fairness can be related to the delay statistics as follow

$$Jain's\ index = \frac{1}{1 + \frac{var(D)}{E[D]^2}} \quad (53)$$

Fig. 12 pictures the Jain index for the same setting as previously. We can see that OCB is less fair than BEB during the transition phase but much more fair in saturation regime. We observe also that during the transition phase, the system can not guarantee equal service time even with the exact window OCB scheme. As the system is not really loaded, neither unloaded, packets got service depending on the system's state at their arrival time: lucky users got immediate service while others are delayed. During the saturation regime, OCB becomes fairer as all packet get access from backoff states while BEB remain unfair due to its intrinsic unfairness.

5 Buffered Terminals Model

In real networks, packets may be queued at node's buffer before being handled by the MAC protocol. It is then necessary to include the queueing delay in the

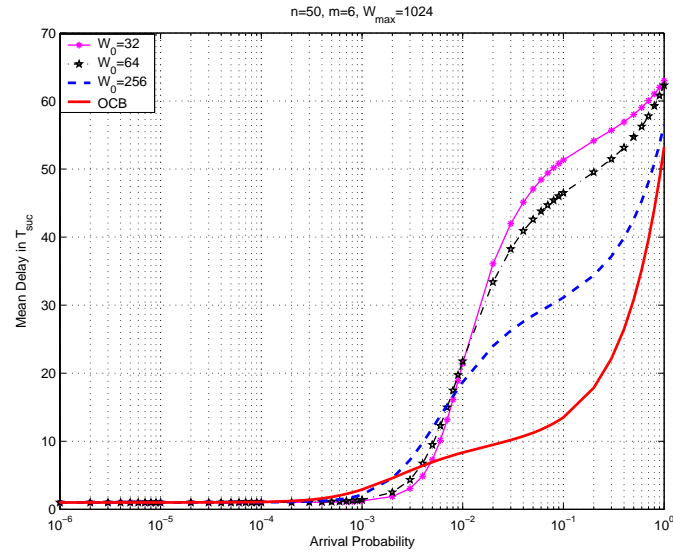


Fig. 11: Normalized delay

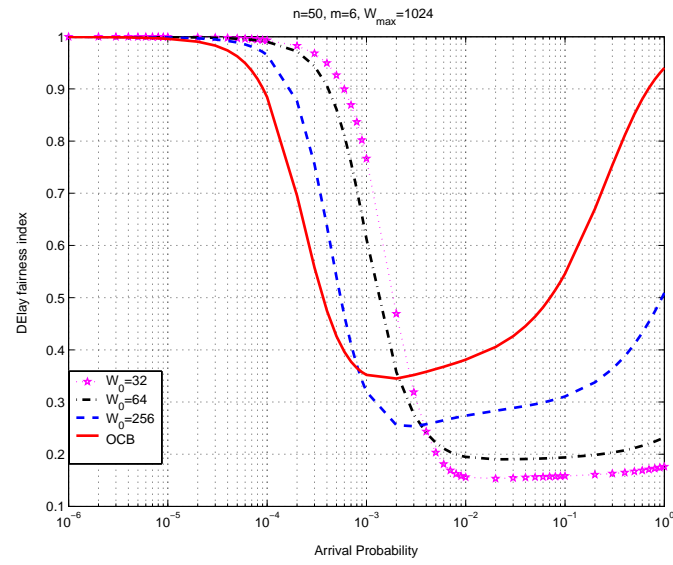


Fig. 12: Jain fairness Index

characterization of the system performance. In section 2.1, we have derived the delay statistics of the protocol for a given packet availability q . We consider now each terminal with a queue of size $(K - 1)$ packets, the probability q corresponds then to the probability that the queue is not empty.

We assume that packet arrivals at each terminal is Poissonian process with mean λ , hence each node buffer can be modeled as a finite capacity single server queue $M/G/1/K$. The number of packet in the system at the embedded points corresponding to the time instants just after a job completion (successful transmission or drop) forms a Markov chain. We define the packet service time as the packet success delay in case of successful transmission or the packet drop delay in the contrary case. The average packet service time at the MAC layer is then

$$\mu = \sum_{i=0}^m D_i^{suc} p_i^{suc} + D_{drop} p_{drop} \quad (54)$$

Let (π_k^d, π_k) denote respectively the steady state probability of having k packets in the queueing system at departure instants, and at arbitrary instants. $k \in \{0 \dots K - 1\}$. And let $Q_{i,j}^d$ denotes the system transition probabilities upon departure, we have then [25]

$$Q_{0,k}^d = \begin{cases} \alpha_k & 0 \leq k \leq K - 2 \\ 1 - \sum_{i=0}^{K-2} \alpha_i & k = K - 1 \end{cases} \quad j = 0 \quad (55)$$

$$Q_{j,k}^d = \begin{cases} \alpha_{k-j+1} & j - 1 \leq k \leq K - 2 \\ 1 - \sum_{i=0}^{K-j-1} \alpha_i & k = K - 1 \end{cases} \quad 1 \leq j \leq K - 1 \quad (56)$$

Where α_k represents the probability of having k arrivals during a service time

$$\alpha_k = \sum_{i=-1}^m \frac{(\lambda D_i^{suc})^k}{k!} e^{-\lambda D_i^{suc}} p_i^{suc} + \frac{(\lambda D^{drop})^k}{k!} e^{-\lambda D^{drop}} p_{drop} \quad (57)$$

The global balance equations and the normalization condition are given as follow

$$\pi_k^d = \sum_{j=0}^{K-1} \pi_k^d Q_{j,k}^d, \quad 1 = \sum_{k=0}^{K-1} \pi_k^d \quad (58)$$

Therefore, the steady state probabilities at arbitrary instants are given by

$$\pi_k = \frac{1}{\pi_0^d + \rho} \pi_k^d, \quad k \in \{0 \dots K - 1\} \quad (59)$$

where $\rho = \lambda\mu$ is the queue load.

The probability of having at least one packet in the queue is then

$$q = 1 - \pi_0 \quad (60)$$

And the blocking probability is

$$\pi_K = 1 - \frac{1}{\pi_0^d + \rho} \quad (61)$$

To specify the service time distribution using results of section 2.1 we need to identify the packet availability probability q . In the same time, to specify the packet arrival probability from the queueing analysis we need to identify the service time distribution!

To resolve this problem, given an input rate λ and a queue length $K - 1$, we use a recursive algorithm to estimate the corresponding arrival probability q .

Starting with an initial guess q_{in} on the arrival probability, we derive the service time distribution, then we use the queueing analysis to identify the produced arrival probability q_{out} (Eq. 60). If the difference between the input probability q_{in} and the output probability q_{out} is greater than a threshold, q_{in} is replaced with q_{out} and the operation is repeated. Otherwise the search is stopped.

Convergence is ensured since the case of $q_{out} > q_{in}$ (respectively $q_{out} < q_{in}$) means that even with a lower estimate of arrival probability q_{in} , and so a lower estimate of the service time, the system is more loaded which indicates that the search must continue on the direction of q_{out} (respectively, even with an upper estimate of the service time the system is less loaded so the search must also continue in the direction of q_{out}).

The average queue length can then be expressed as

$$N = \sum_{k=0}^K k\pi_k \quad (62)$$

The mean packet service time (including MAC delay) by

$$W = \frac{N(\rho + \pi_0^d)}{\lambda} \quad (63)$$

And the end-to-end throughput as

$$Thrp_e = n\lambda(1 - \pi_K)(1 - p_{drop}) \quad (64)$$

6 Simulation results

In this section we validate our analytical results with NS2 (Network Simulator) simulations. We use the same parameters as previously, the queue length is taken equal to 50 and we consider now the RTS/CTS access mode. The optimal constant window in this case is 363 slots. In Fig. 13, we plot the achieved throughput under BEB and OCB schemes vs. data arrival rate. First, we observe that results from analytical model are almost equal to that from simulations which validates, not only our queueing model, but also our non-queueing models and our delay

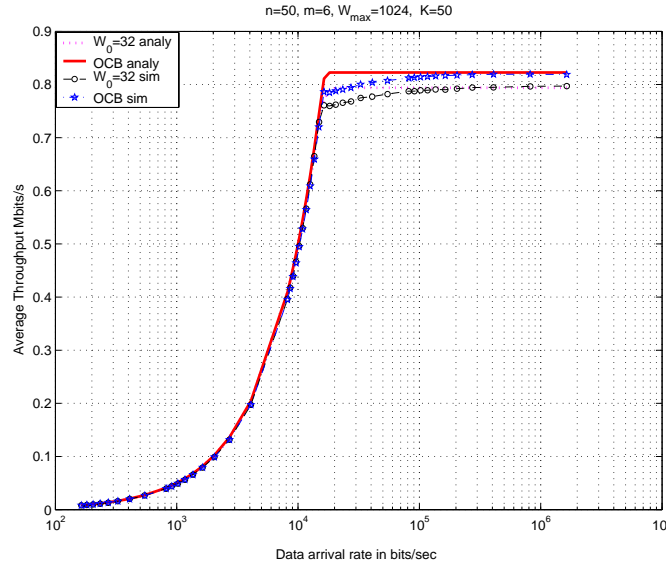


Fig. 13: Queueing model throughput Vs. simulation

statistics model. Second, we can see that BEB performances are close to that of OCB which means that even if BEB collision probability is higher than that of OCB, the penalty in throughput is very small since collision duration is reduced by the use of RTS/CTS handshaking. Fig. 14 depicts the corresponding mean packet service time (including queueing delay) and shows clearly the existence of the three operating modes (no-backoff, transition and saturation regimes). In fig. 15, we plot the delay Jain's fairness index Vs. packet arrival rate. We observe again that during the no-backoff regime the two schemes are fair, at transition regime the two schemes are less fair, and finally at saturation the two schemes becomes again fair which is different from our previous observation when we analyze the delay fairness. This is due to the fact that at saturation, the queueing delay is much more higher than the mac delay. To illustrate the short-term unfairness of the BEB scheme, we plot in fig. 16 the throughput Jain's fairness index using the sliding window method [9]. The data arrival rate is taken equal to 20Kbits/s , the network is then in saturation regime. We observe that OCB is relatively fair even at short time horizon, and is much fairer than BEB.

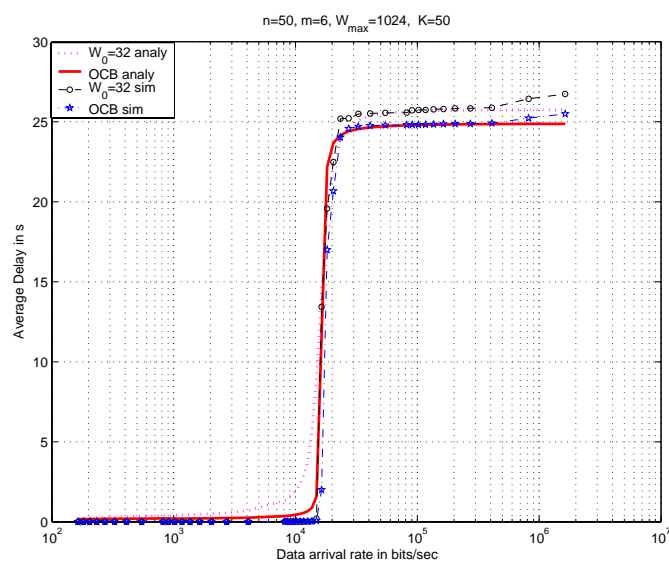


Fig. 14: Queueing model delay Vs. simulation

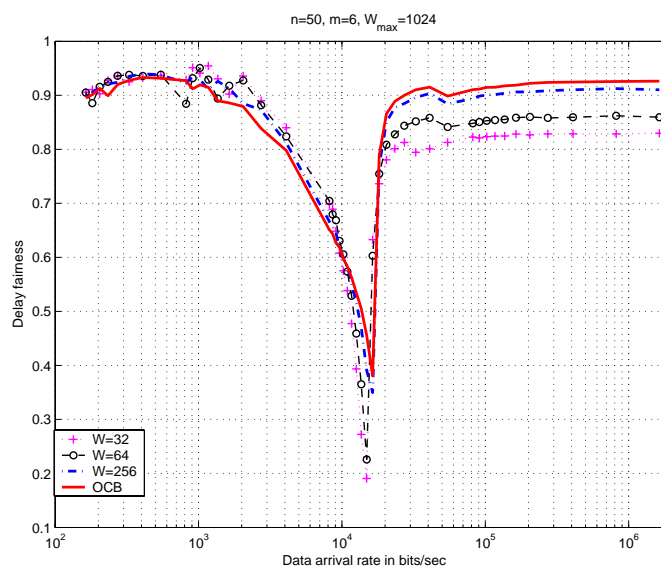


Fig. 15: Delay fairness

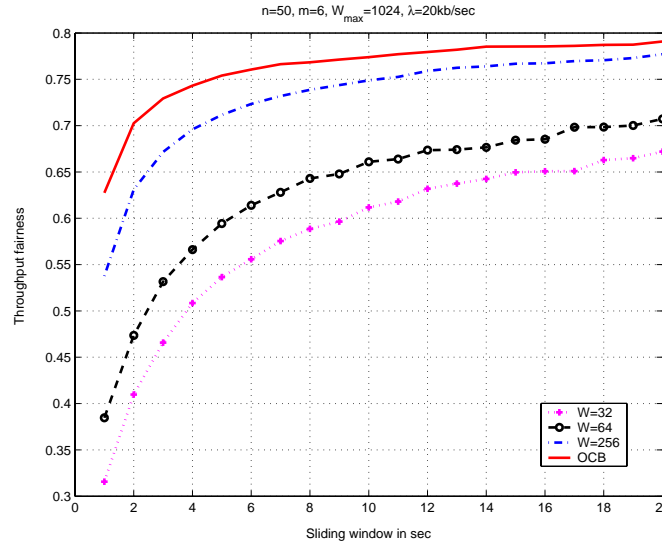


Fig. 16: Throughput short-term fairness

7 Conclusion

In this paper, we investigated the performance of the IEEE 802.11 DCF multiple access scheme under general load conditions in single-hop configurations and we proposed a backoff scheme enhancement that is quasi-optimal under all traffic conditions. First, we presented a Markov chain model to analyze finite load situations without considering queuing at nodes buffer from which we derived an accurate delay statistics model. We derived then the size of the optimal constant window that maximizes the network throughput in saturation regime. Then, we used this window for all traffic loads and we proved that the system operate quasi-optimally independently from the traffic load. Numerical results have shown that OCB performs better than BEB both in term of throughput, delay, and short-term fairness. We have extended then the study to consider finite queueing capacity at nodes buffer, and we have developed a recursive algorithm to alleviate the complexity of the analysis. Finally, we validated our results by NS2 simulations where we show clearly the superiority of OCB over BEB. OCB requires just information about the network size. This information is easier to obtain in single hop networks, and its coherence time is larger compared to other parameters (backlog state, active nodes, or any other information measured from the channel state).

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